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# estimating CdA with a power meter

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# with good data, field testing works

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- a validated power model

Martin, et al. (1998), “Validation of a mathematical model for road cycling power”, J App Biomech 14(3)

- examples with good data collected in field

Martin, et al. (2006a), “Modeling sprint cycling using field-derived parameters and forward integration”, MSSE 38(3):592-597

Martin, et al. (2006b), “Aerodynamic drag area of cyclists determined with field-based measures”, Sportscience 10: 68-9

Snyder, J.; and T. Schmidt (2004), “Determination of drag parameters utilizing a bicycle power meter”, HPeJ issue 1

*... but what happens if the data aren't good?*

# the classic approach

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- constant speed runs on flat windless roads
  - some alternatives: coast down tests, velodrome runs
  - often, results averaged over runs taken in opposite directions
  - occasionally, a few other adjustments and variations
- for constant speed on flat windless roads, power-drag equation simplifies to
$$\text{watts} = k_0 v + k_1 v^3, \text{ or } \text{watts}/v = k_0 + k_1 v^2$$
- so regress drag force (i.e., watts/v) on  $v^2$ 
  - the regression intercept ( $k_0$ ) is related to  $C_{rr}$
  - the regression slope ( $k_1$ ) is related to  $C_d A$

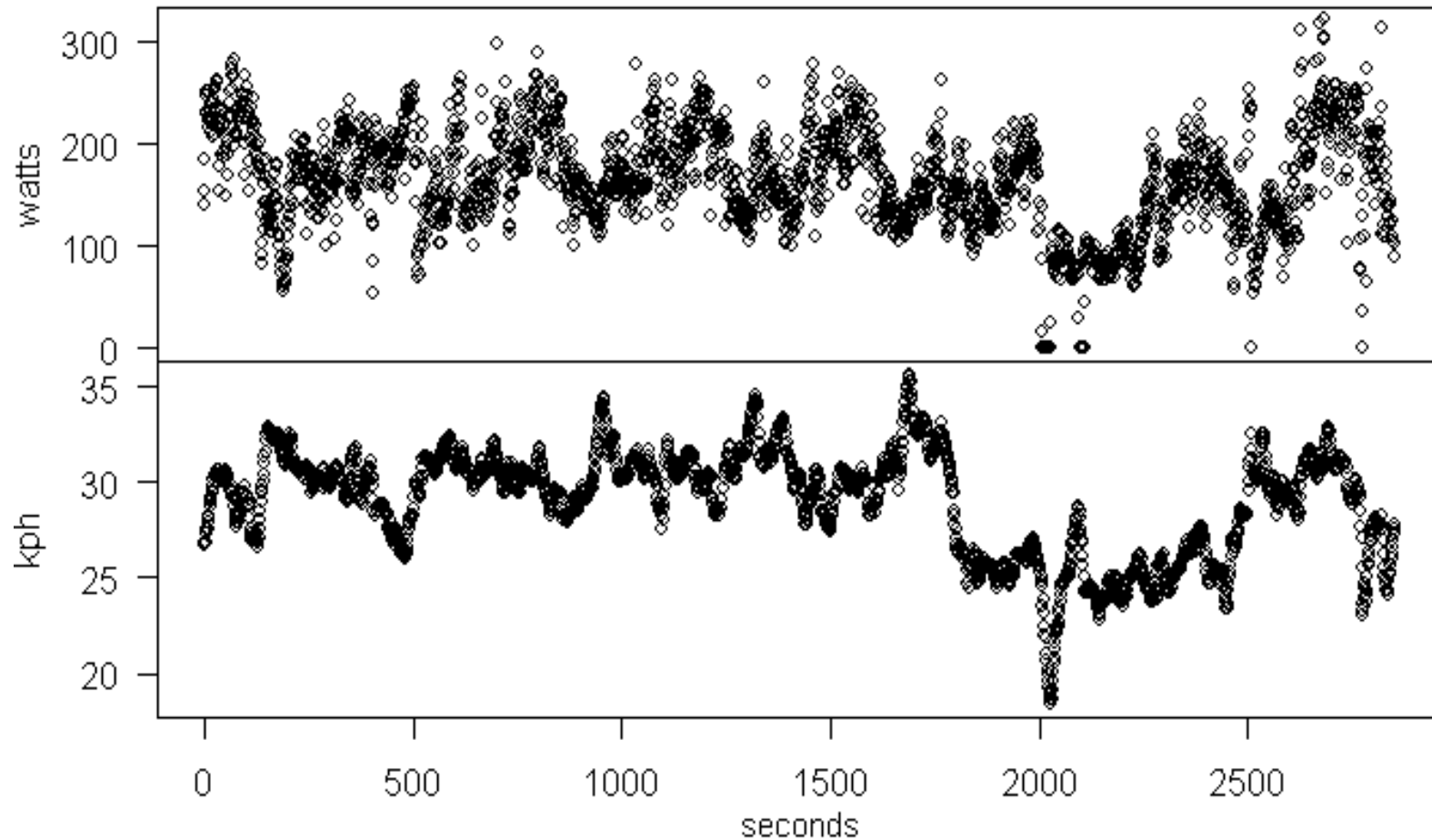
# the challenge

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- I recorded power and speed during a ride consisting of a number of laps around a closed course
  - power was not constant
  - speed was not constant
  - the course was not flat
  - the wind was blowing (weakly but (I believe) consistently and from the same direction during the entire ride)
- how good of an estimate of CdA is it possible to get with these (lousy) data?
  - using usual approach, not good at all
  - using approach described here, not bad at all
  - with non-lousy data, you can get very good results

# the data

data were collected at 1.26-second intervals with a Power Tap hub.  
The plot shows that neither power nor speed were constant

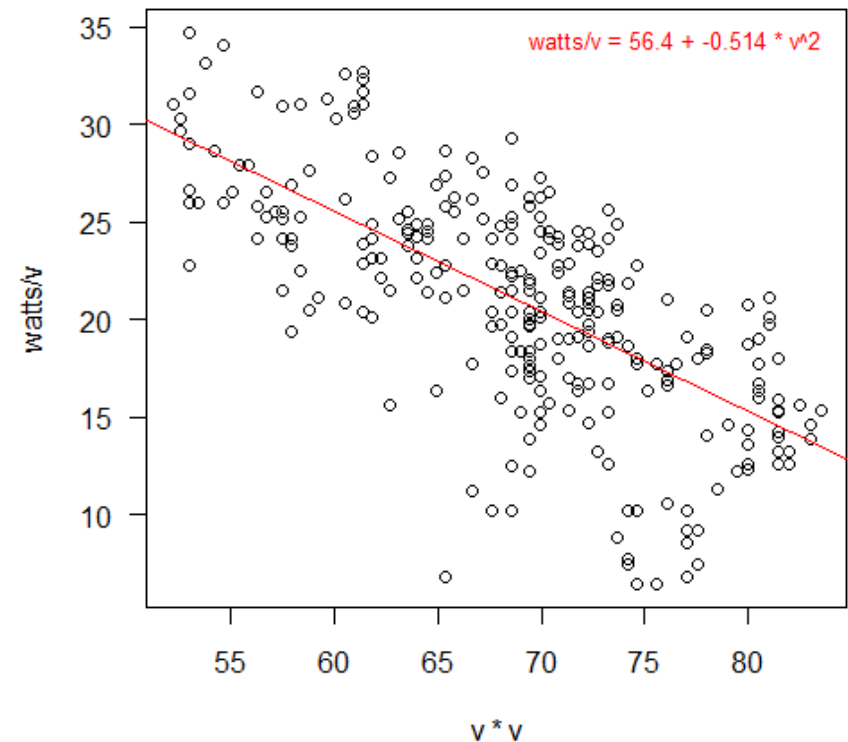


# the problem

- flat, windless venues are hard to find (some use airplane hangars, building hallways, stadium concourses)
- the regression approach is not robust to changing speed, power, or conditions

using a 5 minute subset of the data produces a regression slope that is *negative* (and highly statistically significantly so), meaning negative CdA

**usual methods don't work well with these data at all**



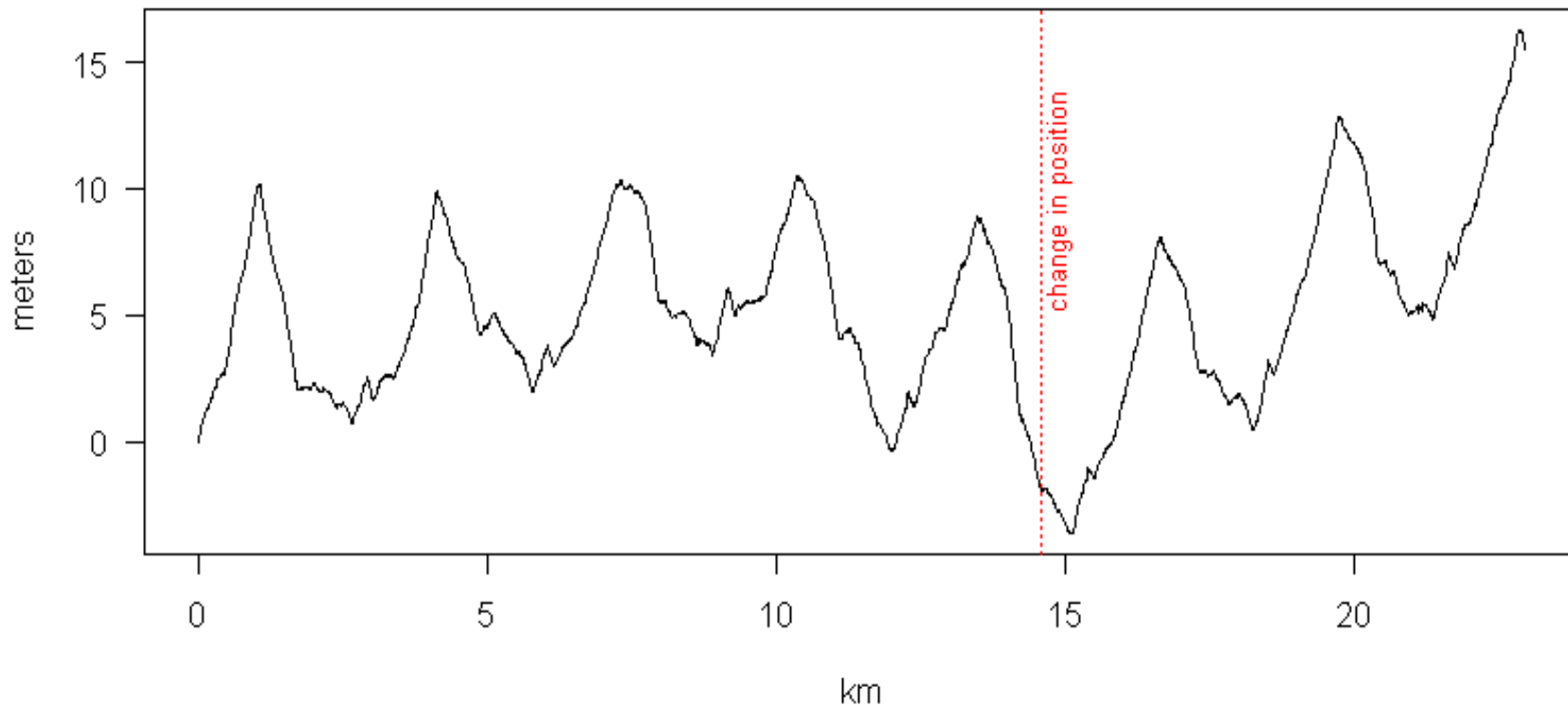
# a different approach

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- record moment-by-moment speed and power from a series of laps on the same route
  - route need not be flat (in fact, it's better if there's some change in elevation)
  - speed and power need not be constant (better to vary speed and power)
  - hold position and don't use brakes
  - wind should be as close to zero as possible
- construct an elevation profile for the ride as a function of known power, speed, mass, and air density, and initial guesses at CdA and Crr. Plot the elevation profile against distance
- since each lap must start and end at the same place, find the value of CdA that produces zero net elevation gain over each lap. One (but not the only) way to do this is to try different values until the laps “line up”

# Q: did we correctly identify laps?

this approach provides a self-check: it should identify the correct number of laps. Analyzing these data reveal seven-and-a-half laps with about 10 meters of elevation change per lap. Was that right?





## A: pretty much, yes

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using *only* speed and power, we identify key features of the ride

correct number of laps? *yes*

correct lap length? *yes* (3.12km)

entered on one side of course and exited on other? *yes*

entered at “bottom” of course and exited at “top”? *yes*

10 meter elevation change over each lap? *close* – I believe it's closer to 15 or 16 meters

correctly identified high and low points within laps? *yes*

shows conditions were not constant (i.e., change of position during last two-and-a-half laps)? *yes*

brief use of brakes on third lap? *yes*

# lap lengths are well identified

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- different guesses about CdA (or Crr or mass or air density) have only a small effect on the estimate of lap lengths
  - changes in the parameters move the curve up-and-down but not left-and-right
- small changes in wind don't affect lap length much so lap lengths are relatively robust
  - however, big changes in wind may
- relatively robust identification of the lap lengths means that it's feasible (though not always wise) to impose the “zero net elevation gain” constraint on each lap
- if you're interested, a Google map of the course is here:  
<http://tinyurl.com/yq9r76>

# why plot?

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- I could have done (and will later show) this algebraically
- however, for now it's easier and perhaps more instructive to plot graphs and show what's happening

algebraic solutions generally look for a parameter that maximizes some overall measure of fit

in this case, you learn more by looking for *specific* areas of fit and misfit rather than *overall* fit

the graphical approach makes it easy to find lap length and knowing lap length will be useful

*perhaps most importantly*, the plots give us a generalizable way to diagnose lousy estimates

# so what was the CdA?

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- hold your horses. First we have to check the assumptions and calculations. To do that, you need to know how to do them.
- we'll start from the beginning, with the power-drag equation, and split the analysis into two parts:

assuming no wind

assuming some wind, but wind which is consistent in speed and direction

# a (simplified) power equation

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$w$  = watts needed to propel bike at speed  $v$

= watts to account for rolling resistance +  
watts to account for change in elevation +  
watts to account for change in speed +  
watts to account for air resistance

$$= W_{rr} + W_{PE} + W_{KE} + W_{aero}$$

# simplified power equation, continued

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$$W = W_{rr} + W_{PE} + W_{KE} + W_{aero}$$
$$= C_{rr} m g v + s m g v + m a v + \frac{1}{2} C_d A \rho v_{air}^2 v$$

where

$v$  = speed in m/s (i.e., “ground” speed)

$m$  = total mass (kg) of rider + bike

$g = 9.81 \text{ m/sec}^2$

$C_{rr}$  = coefficient of rolling resistance

$s$  = slope

$a$  = acceleration

$\rho$  = air density

$v_{air}$  = “air” speed of bike

$C_d A$  = drag area

# no wind approach

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- assume  $v_{\text{air}} = v$  and solve for slope as a function of other variables

$$s = w/(m g v) - C_{rr} - a/g - (\rho C_d A v^2)/(2 m g)$$

- use this formula to estimate point-by-point slopes from the data, supplemented by initial guesses at  $C_{rr}$  and  $C_d A$ . Ballpark guesses for starting values might be  $C_{rr} = .005$  and  $C_d A = 0.3$ .  $v$  is in meters per second, so convert  $v = \text{kph}/3.6$ . Since these data were collected at 1.26-second intervals a reasonable estimator for  $a(t)$ , the acceleration at time  $t$ , is  $(v(t+1.26) - v(t-1.26))/(2 * 1.26)$

standard approach assumes accelerations = 0. This approach calculates and uses them

- use estimated slopes to construct elevation change for each 1.26-second interval:  $\text{elev.change} \approx s * v * 1.26$

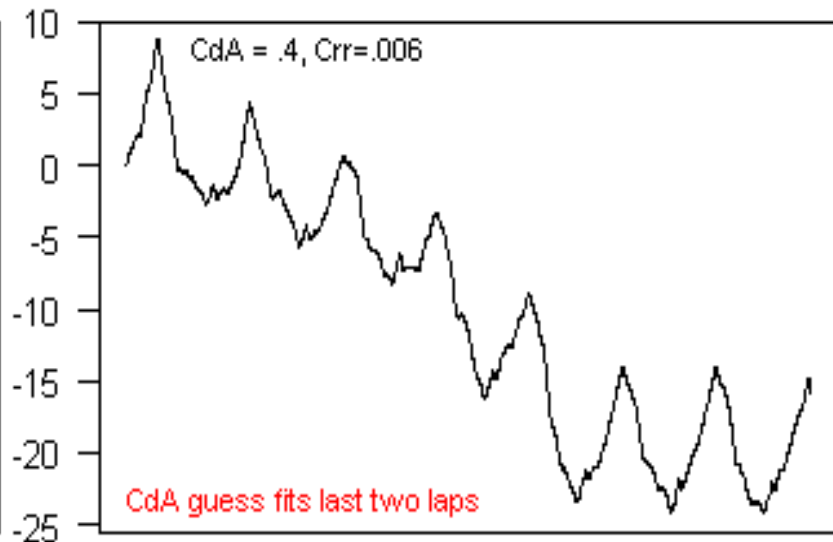
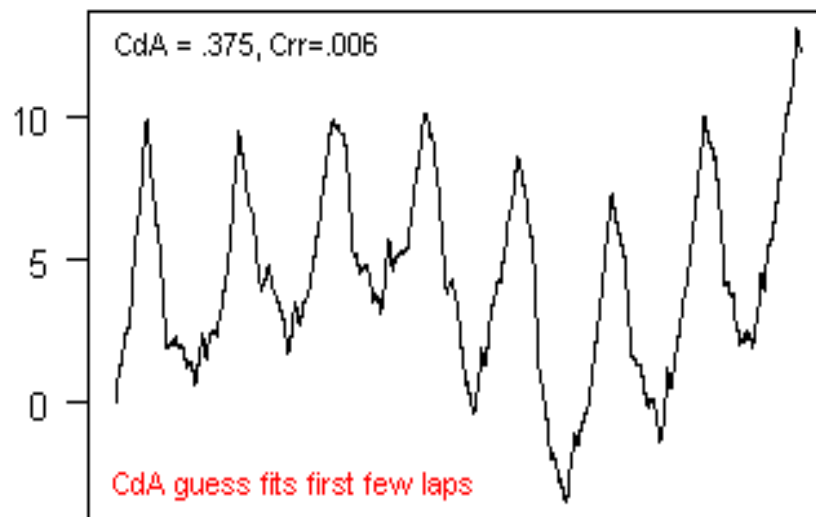
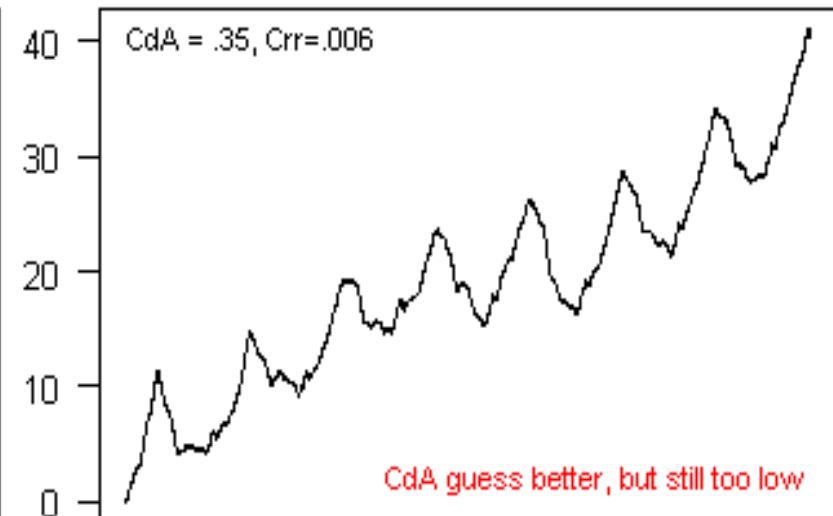
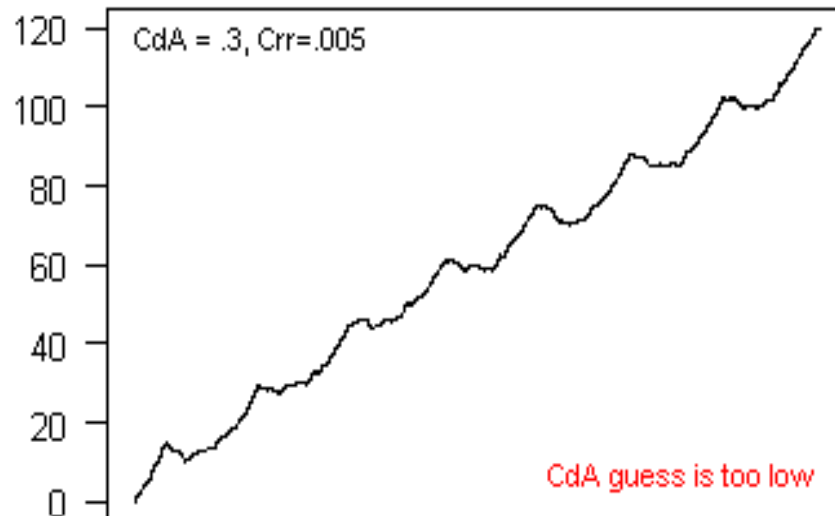
# produce an “elevation” profile

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- cumulate the “elevation” changes and plot against distance to produce a “virtual elevation” profile
- later we'll see how to solve algebraically for the CdA that makes the plotted laps line up but for now just make guesses at CdA to achieve that
  - the advantage of making iterative guesses at the CdA and re-plotting is that you can see whether the elevation profiles are well-behaved and learn how sensitive or robust the CdA estimates are
- these steps may seem daunting but they only take a few commands in any spreadsheet or programming environment like R or Matlab, and have already been implemented in Golden Cheetah, the open-source cycling analysis program



# estimated CdA should level the profile



# why does this work?

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classic approach regresses average drag force on  $(\text{avg } v)^2$  and minimizes sum of squared errors. Instead, we minimize the sum of a more complex form of the error: we integrate the moment-by-moment elevation changes across distance, then impose a new constraint that the elevation gain across laps must net to zero

laps are extra information that the classic approach ignores. In addition, the data are sequential. Sequencing the data means accelerations can be calculated and included instead of assuming they are zero. There are other ways to produce a “solution system” but the elevation profile is a convenient way to maintain the sequence of the data, to allow for the additional constraint on fit, and to provide a diagnostic of specific areas of fit and misfit

# when doesn't this work?

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- this method models moment-by-moment power as a function of moment-by-moment speed and changes in speed but *everything else* gets tossed into a residual that we then model as if it were a slope. That's why what we get is a “virtual” elevation profile
- if 1) there are errors in measurement, or 2) the unmodeled parts of the power equation (like wind or brake usage) are large relative to the modeled parts, or 3) CdA changes because you didn't hold your position, or 4) you're losing air in your tires so the Crr is changing systematically, then the virtual elevation profile will differ from the true elevation profile

we'll see more discussion of this when we talk about wind

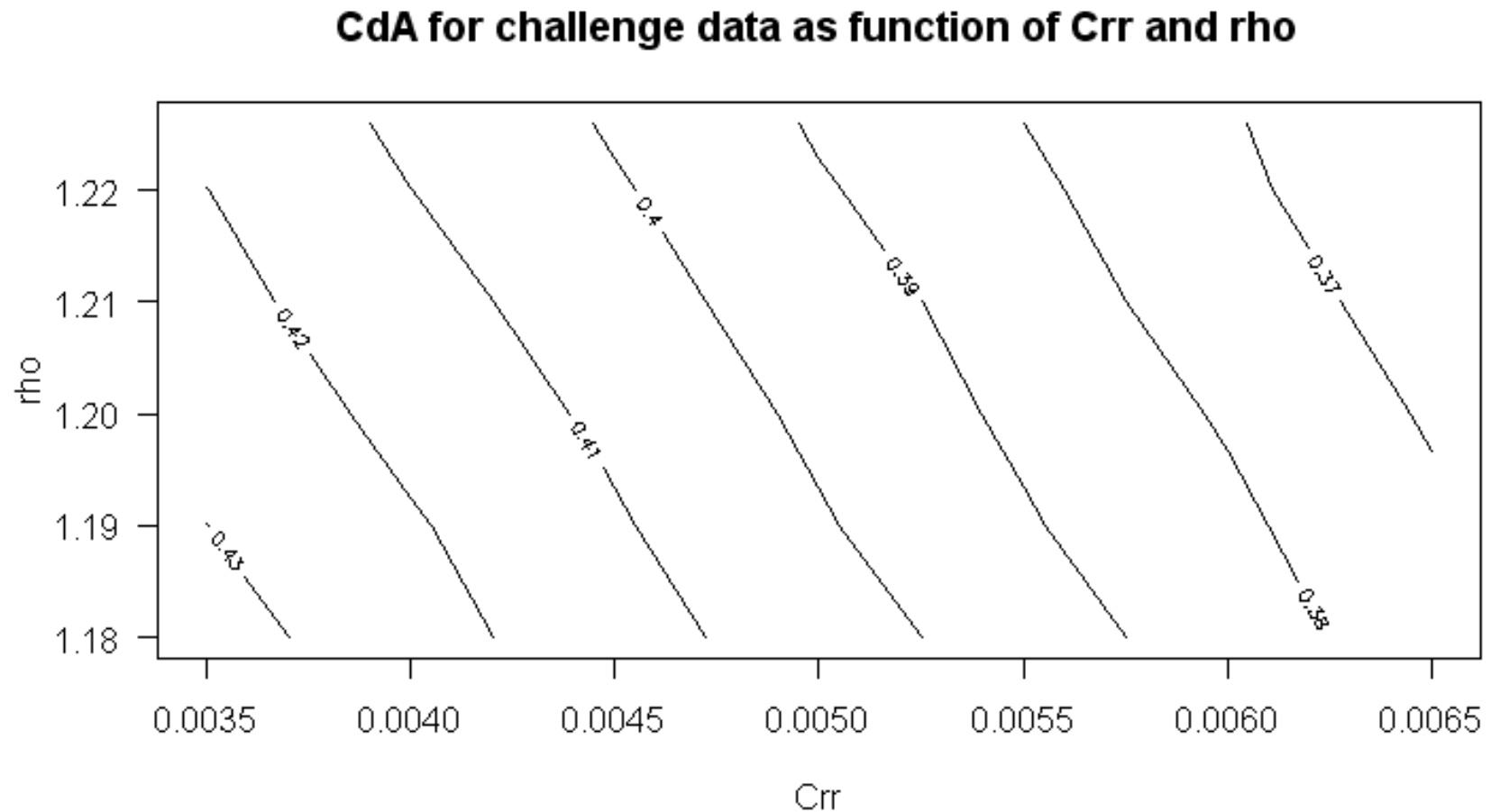
# absolute and relative CdA

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- in this example I made guesses about both CdA and Crr. Look at the equation—Crr and slope have the same “scale” so an increase of .001 in Crr looks like an increase in the slope of .001 (=0.1%). Later this will turn out to be handy.
- Nonetheless, *with these data*, we appear to have pretty good relative accuracy but unless we know what Crr is, we won't have good absolute accuracy
  - good relative accuracy means we can spot small *changes* in CdA even if (with these data) we can't nail down CdA itself. Sometimes you'll want do specific additional tests that will let you nail down both CdA and Crr
- I was bad and didn't measure air density (though I have a ballpark idea about what it was)

# so what was CdA?

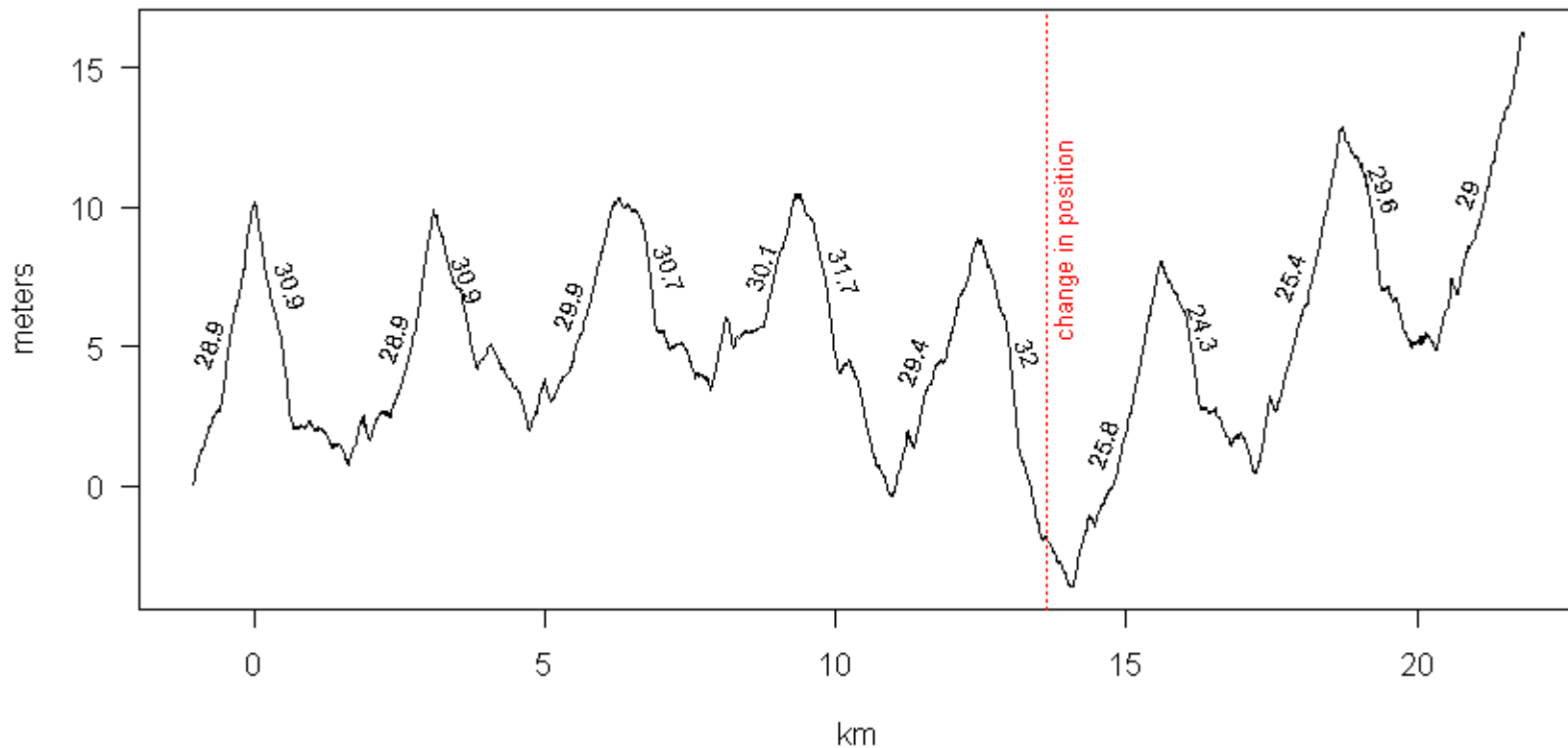
since there were so many things I didn't record, the best we can do *with these data* is to calculate CdA using a rough guess at my total mass and assuming different values of Crr and air density. We get:



# what about lap amplitude?

you may have noticed that the “amplitude” of the estimated elevation differed across laps. Could it be related to speed?

kph and estimated elevation profile



# ground speed and air speed

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- there is rough evidence that for these data the elevation profiles are speed-dependent
  - increased speed in the downhill direction increased elevation change
  - increased speed in uphill direction decreased elevation change
- could it be unmeasured wind?
  - up to this point, we've assumed no wind (i.e., ground speed = air speed)
    - recall that the challenge included the information that there was an unmeasured amount of wind, but that I thought it was light and from a consistent direction

# a handy diagnostic

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now you can see why we started with a graphical approach even though we could have solved algebraically for an exact solution: the graphical approach provides a handy diagnostic for whether the model assumptions are met

unmeasured variables affect the profiles in recognizable ways

unmeasured wind typically makes the profiles speed dependent

unmeasured braking typically appears as a sudden jump in the estimated elevation

for an out-and-back the outbound and inbound profiles won't match



# what about the wind?

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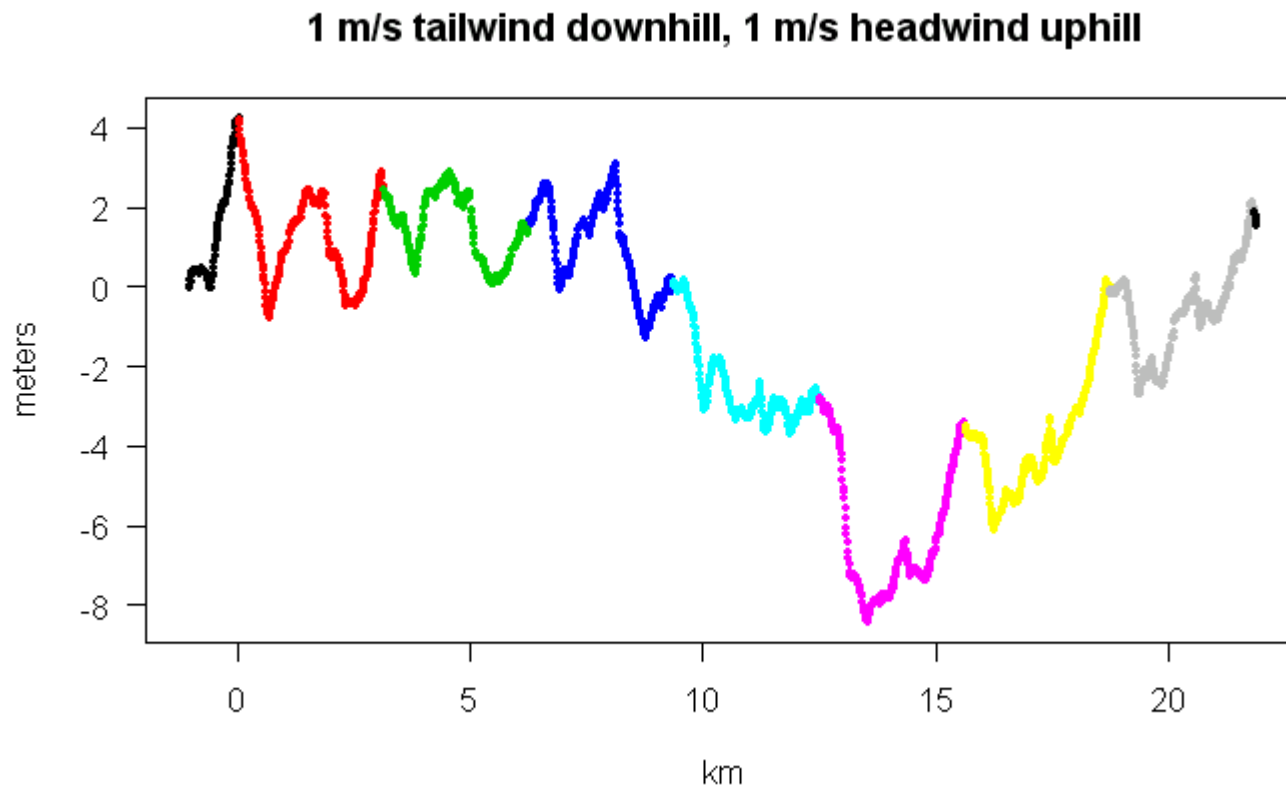
is it possible to say anything about the wind from the data we have?

we'll try adding a (small) non-zero tailwind for the downhill direction and an equivalent headwind for the uphill; then switch

note that this is only a rough correction: the actual course was not a straight out-and-back so adding a small amount of tailwind and headwind is a simplification – the actual course was closer to a right triangle. In addition, roughly half of each of the “downhill” and “uphill” sides were exposed to the wind while the other halves were moderately protected

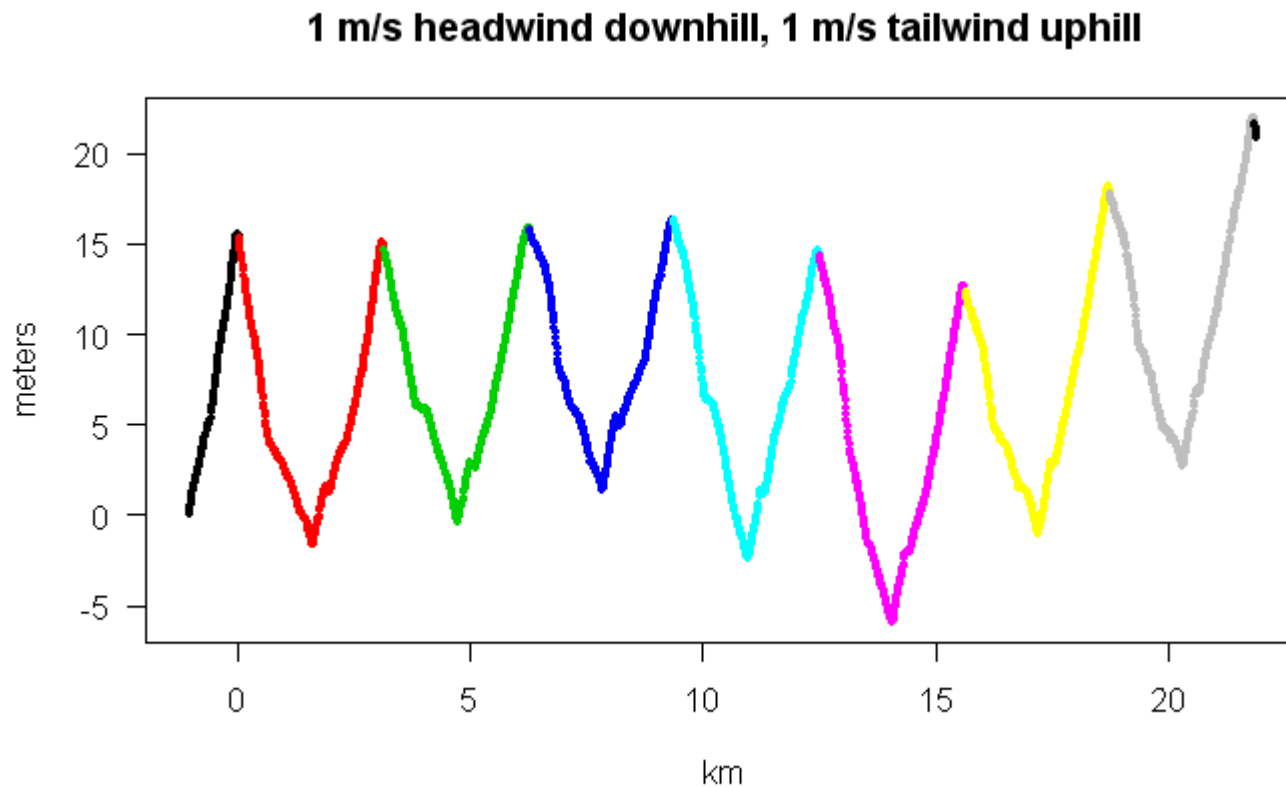
# downhill tailwind, 1 m/s

here's a new estimated profile, assuming a consistent 1 m/s tailwind in the downhill segment and a 1 m/s headwind in the uphill segment. Notice that the laps don't have the same shape



# downhill headwind, 1 m/s

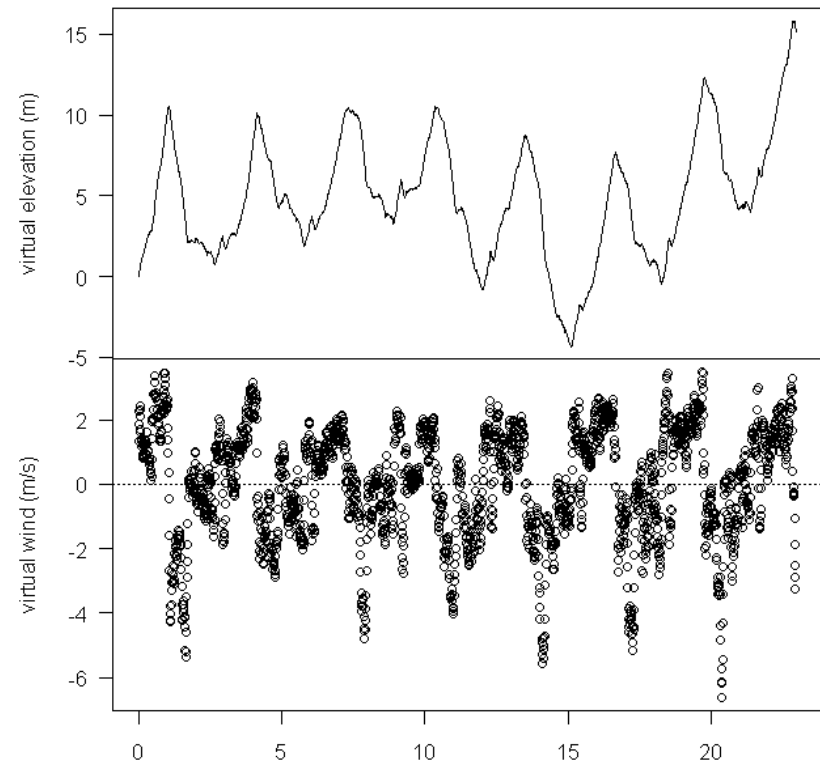
the lap amplitudes and profiles are much closer, and total elevation gain over each lap appears to be around 16 meters



# what about virtual wind?

- virtual elevation assumed zero wind. For virtual wind assume an elevation profile and see what the wind must have been
- can you see change in position for last two-and-a-half laps in the virtual wind plot, or the slight use of brakes at “top” of lap 3? (neither can I).
- virtual wind is much noisier than virtual elevation

that's why we model on  
VE rather than VW



# will it work with other examples?

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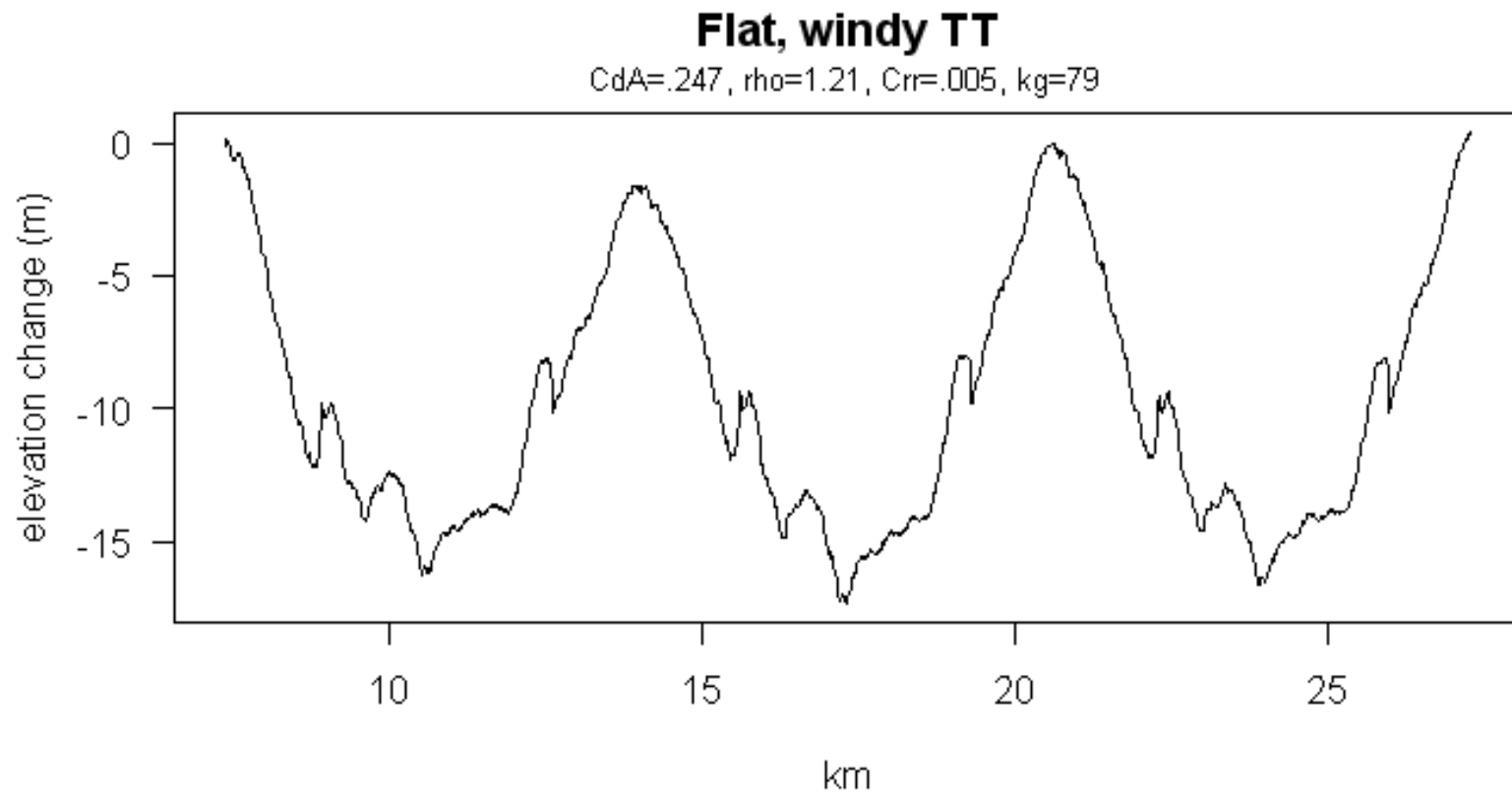
- would I be showing these to you if I thought it didn't?
- when  $C_{rr}$  is known this method matches wind tunnel and classic field tests to within  $\pm 1\%$
- the following three examples illustrate the method with data not collected by me

three laps at Fiesta Island under windy conditions

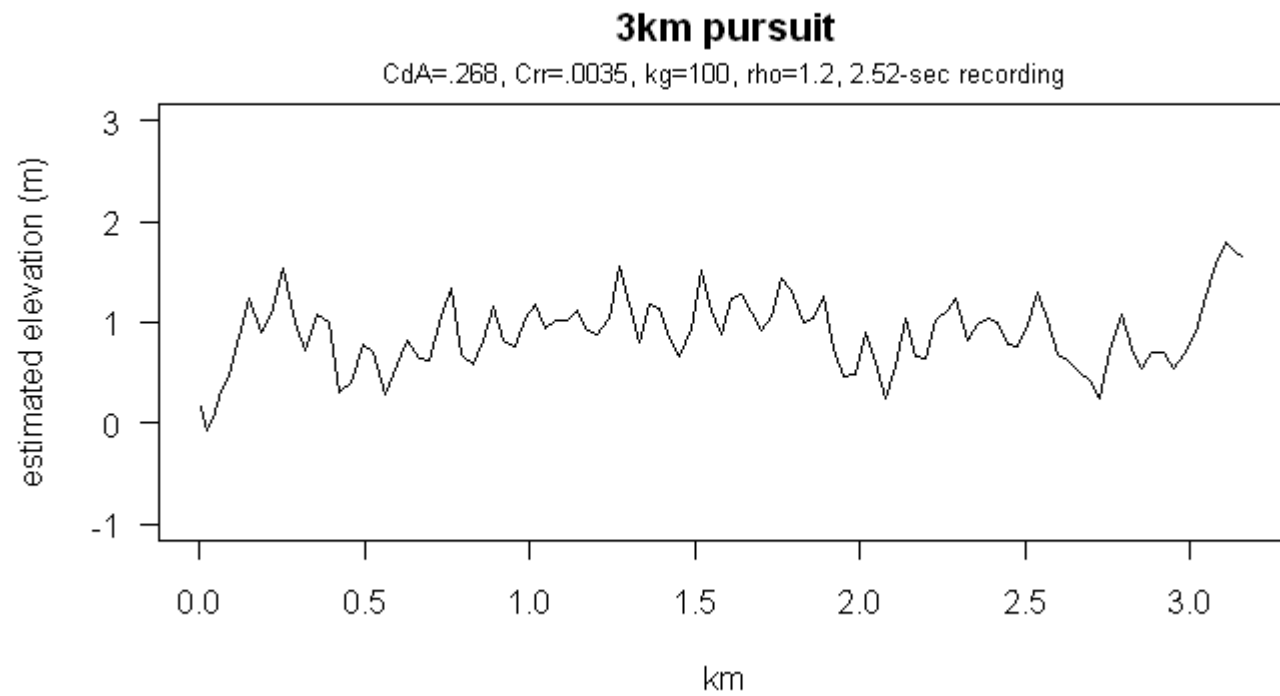
a (flat, windless) race on the track

Dede Demet's Montreal World Cup win: a hilly road race

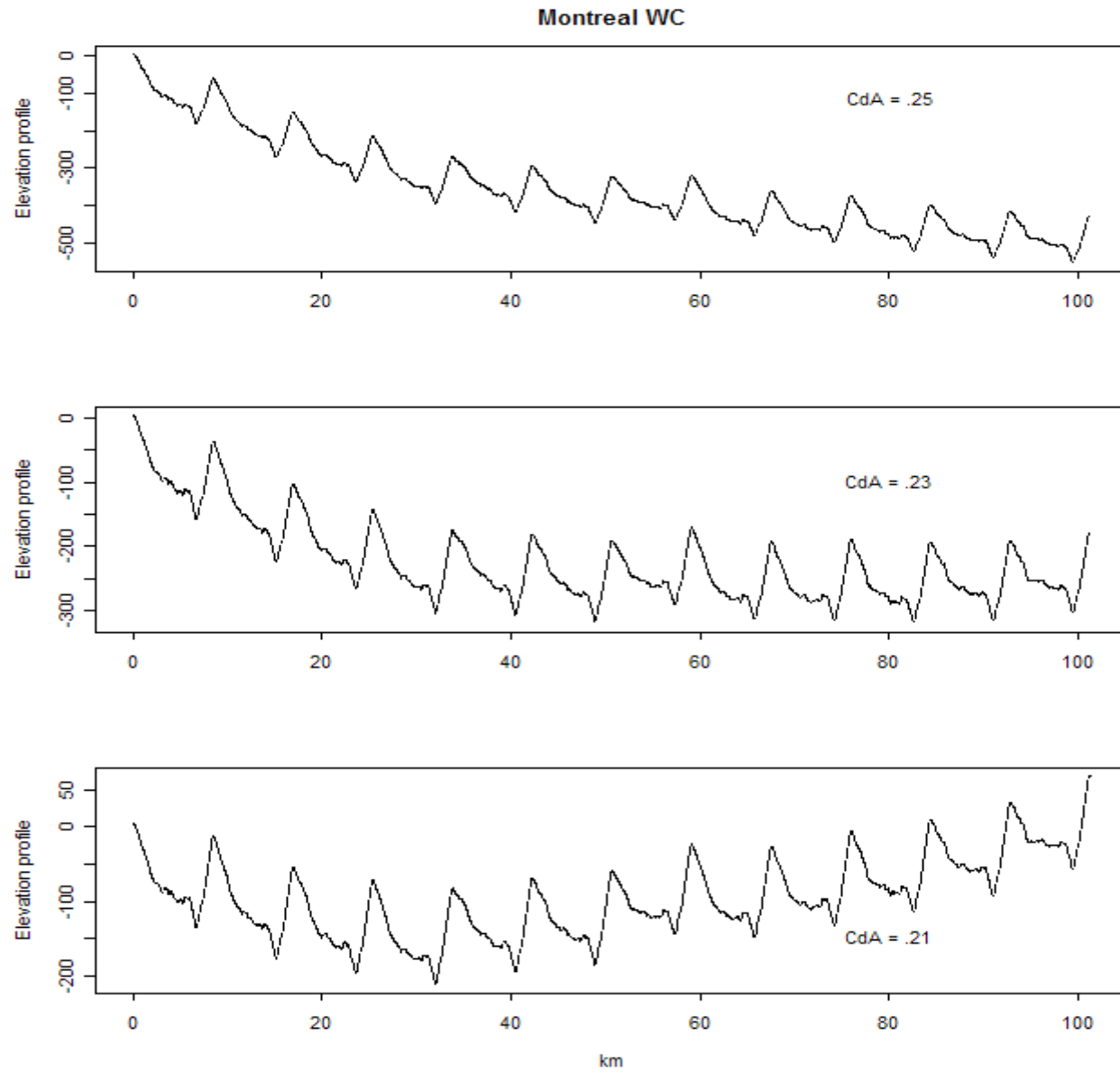
# flat TT



# track race



# world cup road race





# virtual altimeter

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the next two examples show that the method appears to be both reasonably accurate and consistent

## up and down Palomar Mountain

black shows altitude as reported by an altimeter, red shows estimated altitude, emphasizing that braking is ignored

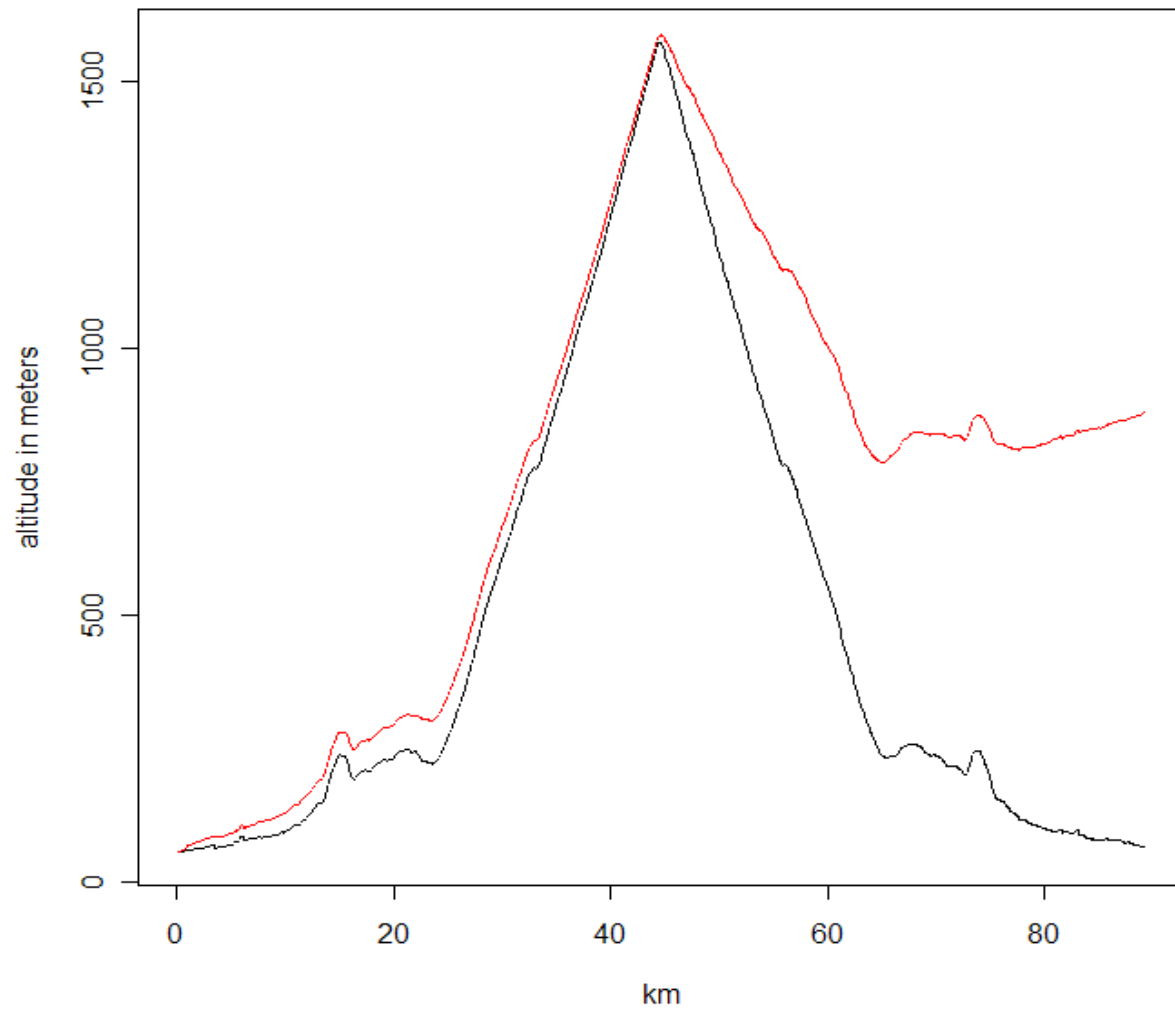
## two years of the San Bruno Hill Climb

elevation profiles calculated from power and speed files for the same rider in two consecutive years, showing that the method captures consistent features of the profile

# palomar mountain

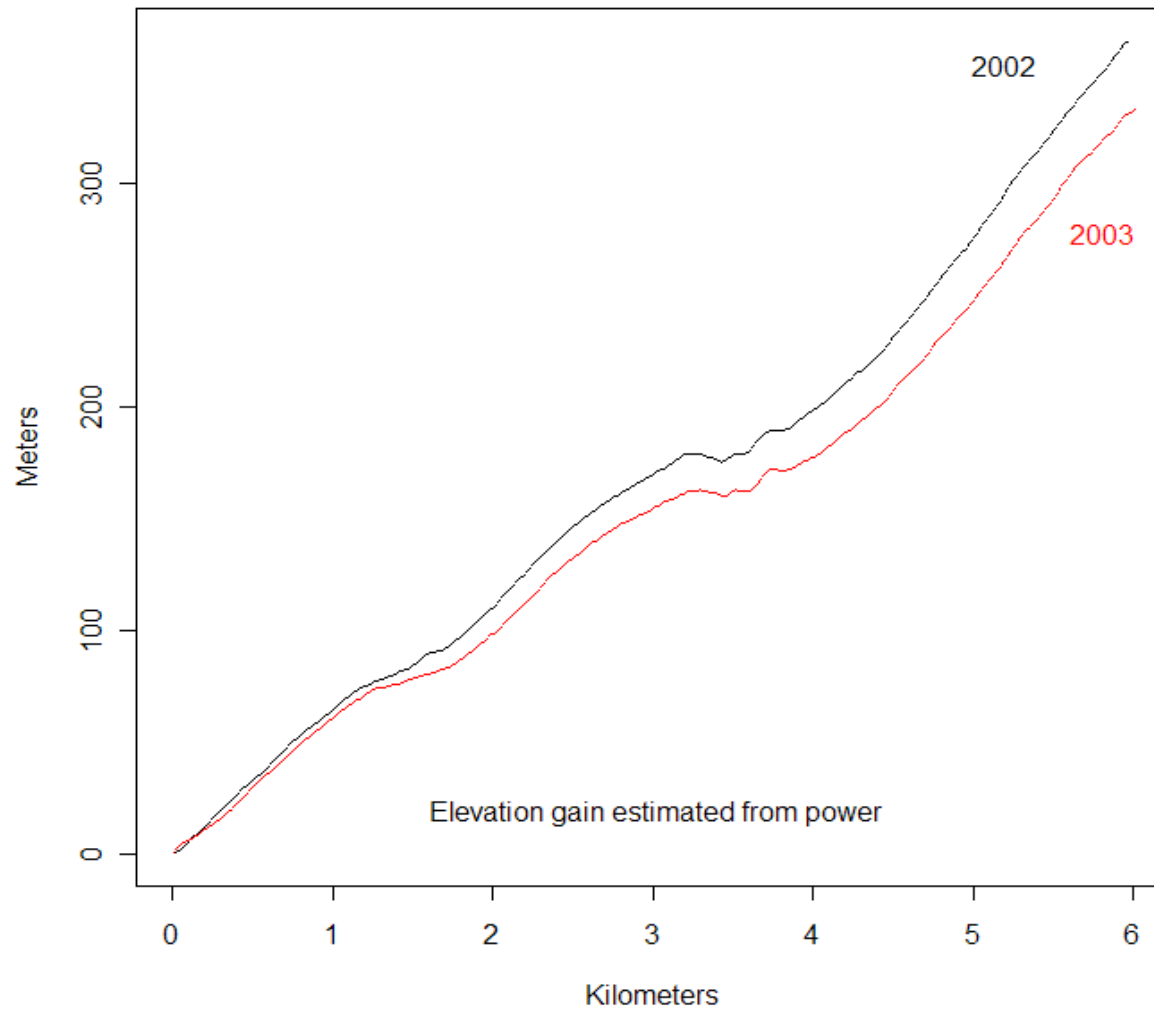
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Palomar profile based on altimeter and power data



# san bruno hill climb

Estimated elevation gain, San Bruno Hillclimb



## other examples

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- growing number of examples of this method being applied by other people on their own data
  - lots of flexibility on venue: out-and-backs, especially for “U”-shaped courses can be good
- spreadsheets exist to simplify calculation and an implementation is available in Golden Cheetah

# functional precision

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- with practice, riders have reported consistently being able to distinguish small changes in position and equipment (like helmets, frames, or front brakes)
- as already discussed, VE profiles reveal previously undisclosed events like the momentary use of brakes or a change in position
  - riders have reported detecting a change when another rider enters a velodrome on the opposite side of the track, or a hill graded not to spec, or a client who had changed saddle height without informing his coach
  - This is a strong result:** VE profiles are functionally precise enough to make reliable predictions that are subsequently verified as true

# statistical precision

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- with careful experimental control by expert users, classic field testing has produced CVs on the estimate of CdA as low as 1.3% (e.g.,  $sd = .0025 \text{ m}^2$  on an estimated CdA of around  $.2 \text{ m}^2$ )
- direct assessment of the variability of VE-based estimates of CdA by “similarly” experienced users has produced CVs as low as 0.3% (i.e.,  $sd = .0006 \text{ m}^2$  on an estimated CdA of around  $.2 \text{ m}^2$ )

# do we need to know $C_{rr}$ ?

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- often, we don't
  - often (though not always) we're more interested in knowing how a change in position or equipment affects  $C_dA$ . If we test using the same tires and tubes on the same roads on the same day at the same pressure then  $C_{rr}$  is a constant and we can concentrate on estimating changes in  $C_dA$
- sometimes, we do
  - in those cases, we need a way to estimate both  $C_dA$  and  $C_{rr}$

# prying apart CdA and Crr

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- we've been starting with a guess at Crr. Is it possible to estimate Crr separately?

sometimes, yes

- first, understand why we've needed to do this

the usual field test approach requires flat roads (or constant grade). The approach described here allows you to use roads that aren't flat so we've lost a constraint. We need some extra information to compensate.

one possibility: make known changes and repeat

another: know the true elevation profile



# extra information demands extra care

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- requires very careful test runs
  - testing on a calm day helps a lot
  - knowing true elevation profile helps a lot
- two examples
  - same hill, different speeds
  - flat course, monotonic speed change (aka the Shen method)

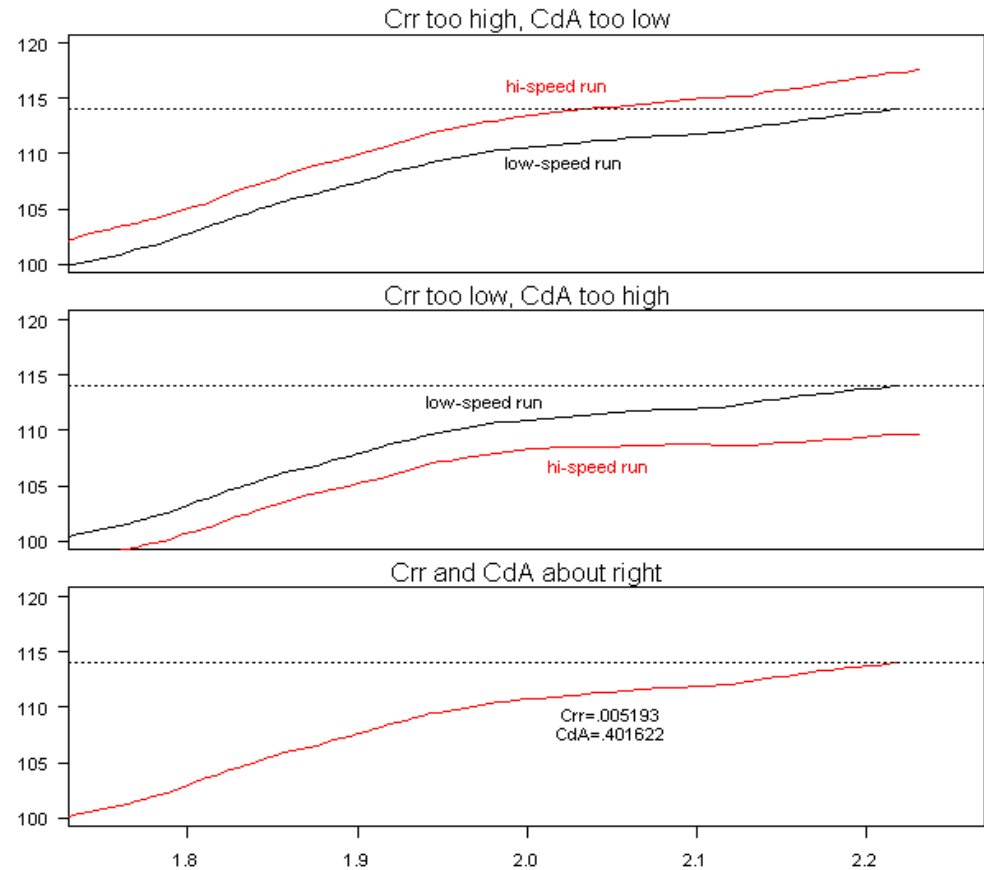
# same hill, different speeds

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- I rode up the same hill twice: once slow and once fast(er)
  - first ~ 170 watts, second ~ 245 watts
  - from topo maps climb known to be 114 meters
  - (this time) checked weight and air density
  - almost no wind
- thus far, we have only been solving for (Crr, CdA) pairs
  - if Crr overestimated then calculated CdA will be too low
  - if Crr underestimated then calculated CdA will be too high
  - more importantly, each (Crr, CdA) pair implies a different amount of total climbing
- knowing total elevation gain adds another constraint

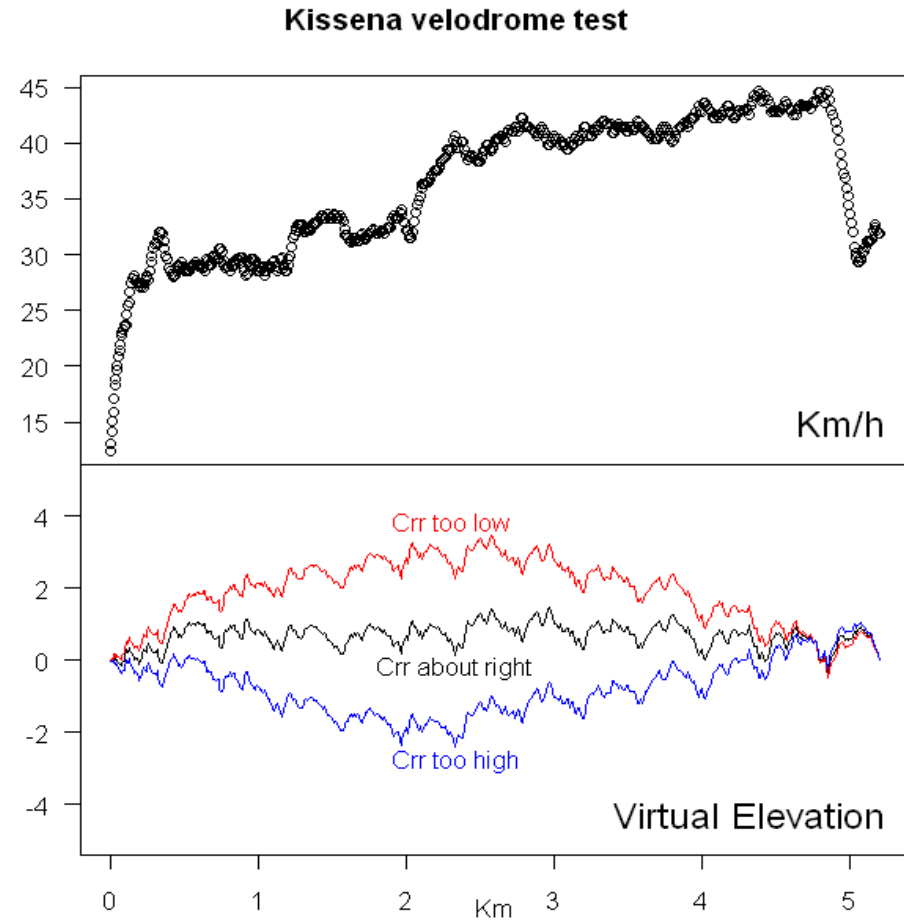
# Crr and CdA constrain elevation gain

- for a given Crr and speed, you can always find a CdA that matches a given total elevation change
- but for two different speeds there is only one (Crr, CdA) pairing that matches a given total elevation change at both speeds. That's when the red and black lines coincide



# flat course, monotonic speed change

- increasing speed on a velodrome
- only one ( $C_{rr}$ ,  $C_dA$ ) pairing flattens both the overall profile and each of the individual lap profiles



# can we generalize?

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- Crr and CdA constrain total elevation gain – *but they also constrain elevation gain over any segment*
- if we know true elevation profile over the entire course we can fit to *arbitrary* segments
  - this can come in handy for velodrome laps since we know the true profile
  - be wary of using GPS altitude readings as the “true” elevation profile—consumer GPS units are accurate but may not be precise enough
  - be wary of using barometric altimeter readings as the “true” elevation profile—barometric altimeters are precise but may not be accurate enough

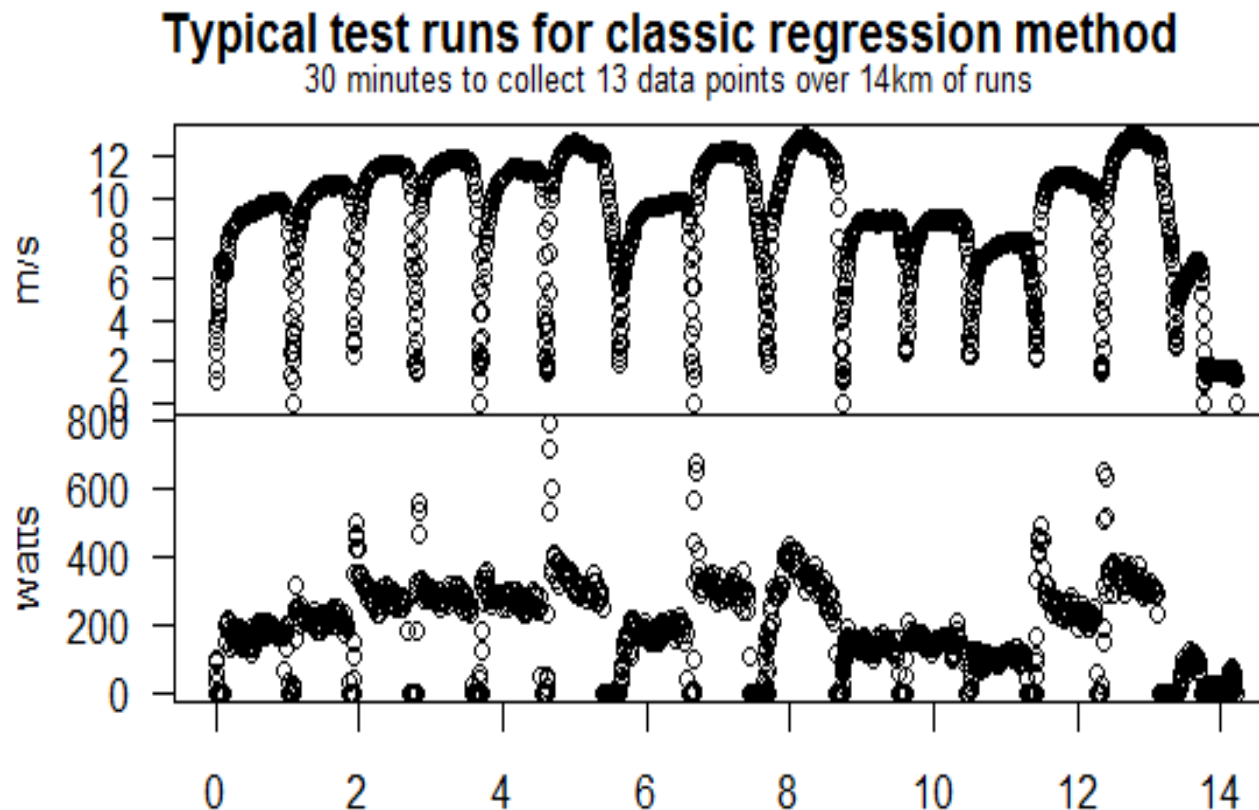
# twice around the block

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- not yet convinced? Try a quick “twice around the block” run  
find a nearby block that lets you make the turns **safely** (both for you and for others) without using the brakes. Start the interval recorder, ride around the block with your hands on the brake hoods then, still without braking, move your hands to the drops and ride around the block again. You can vary your speed as long as you can make the turns without hitting the brakes. Just hold your position and line. Then end the interval
- even with this crude test, as long as the wind isn't too terribly bad you ought to be able to see the change in CdA
- a couple of "twice around the block" trials will give you good insight into what you'd need to do to improve precision

# revisiting the classic protocol

- classic protocol uses multiple runs on flat wind-free venues each at a different speed. Speed and power might look like this:



# improving the classic protocol

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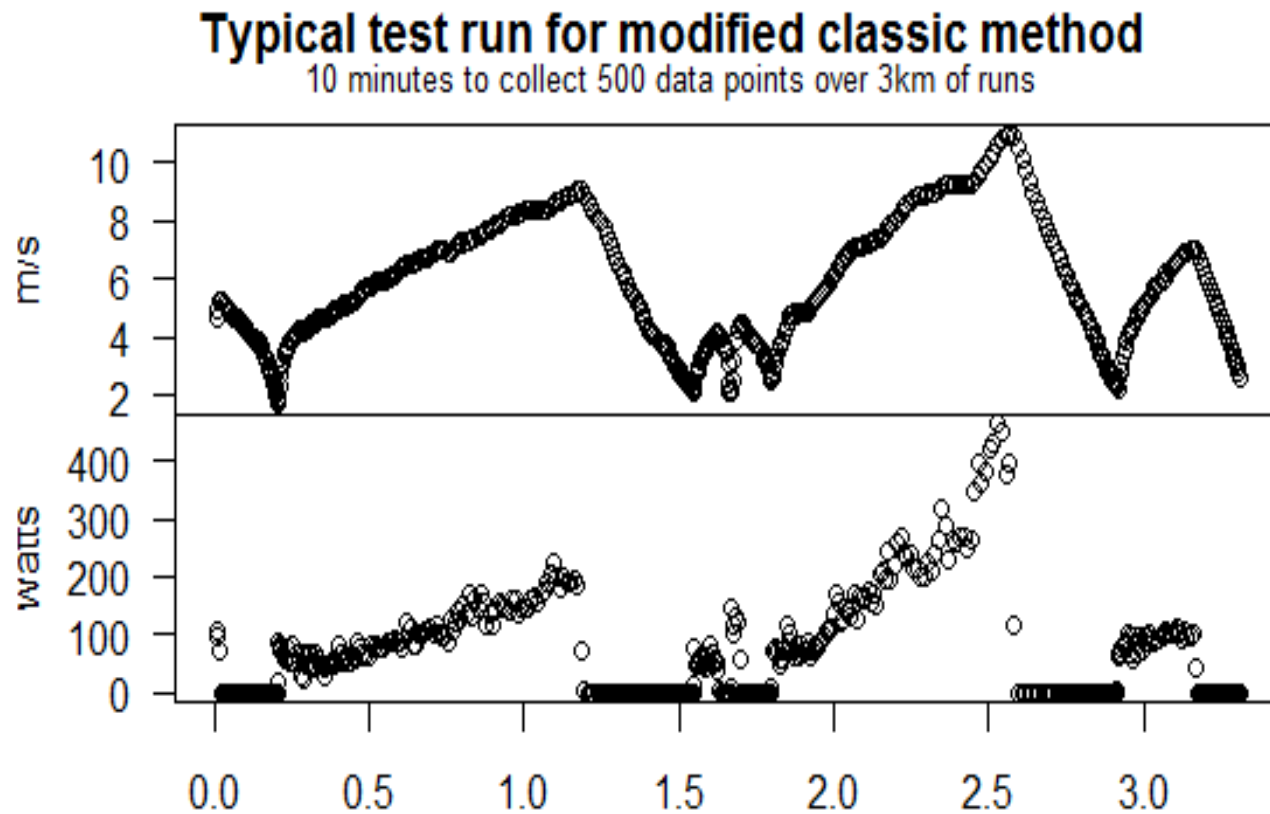
- instead, slowly and steadily increase from a stop up to your cruising TT speed, then coast down to 1 or 2 m/s. Repeat for inbound direction
- since we now know how to handle accelerations, calculate adjusted moment-by-moment power:  $w = \text{watts} - \text{watts}_{\text{KE}}$
- regress  $w$  on  $v$  and  $v^3$

this cuts the number of runs down which saves time while improving precision since sample size and resolution increases



# faster, shorter, better

- speed and power might look like this:



# estimating CdA without a power meter

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- coast downs are common method to estimate CdA
  - some compare top speed, others compare rollout
  - some use hills, others use flat roads
- usual coast downs have poor precision
- can get improved estimates with good speed recording
  - VE is based on power and speed; in coast downs you know power is zero so record speed with a data logger and proceed as usual
- possible to do this with bikes, cars, soapbox racers, racing wheelchairs

# applications

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- racing wheelchairs

in 2007 UIUC Dept of Aero Eng sponsored wheelchair racing club and performed aero drag tests in concourses underneath football stadium using conventional methods. Poor precision because floor was uneven. High-precision speed recording combined with VE could produce better results

- soapbox derby racers, go karts, drivetrain losses, etc.

Martin et al. (2006) presumes fixed drivetrain loss of 2.3%. VE could be used on a bike equipped with both a SRM and PT: find the difference in recorded powers that makes SRM and PT VE profiles identical

# shallow hill coastdowns

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- here are data from two coastdowns I did one afternoon on a shallow hill. The speeds are in km/h, at one second intervals  
run 1: 15.2, 15.8, 16.3, 16.9, 16.9, 17.4, 17.7, 18.1, 18.5, 18.9, 18.6, 17.9, 17.9, 18.6, 19.7, 20.0, 20.9, 21.6, 22.5, 22.5, 23.4, 23.7, 23.5, 24.3, 25.1, 25.7, 26.0, 25.6, 25.3, 24.6, 24.2, 23.9, 23.5, 23.1, 22.9, 22.6, 22.3, 22.3, 22.0, 21.9, 21.7, 21.8, 21.4, 20.7, 20.9  
run 2: 26.6, 26.6, 26.6, 26.5, 26.6, 26.8, 26.1, 25.4, 25.6, 26.3, 26.8, 27.3, 27.9, 28.4, 29.1, 28.7, 29.6, 29.8, 30.6, 30.3, 29.7, 29.2, 28.7, 28.3, 27.4, 27.2, 26.7, 26.1, 25.8, 25.5, 25.3, 25.0, 24.5, 24.2, 23.9
- coast downs = zero power. Let's assume total mass = 86 kg,  $\rho = 1.17 \text{ kg/m}^3$ , and total drop from entry to exit of the test section was exactly 5 meters. Can you estimate  $C_d A$  and  $C_{rr}$  from these data?

# summary

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- knowing only speed and power still provides an impressive amount of information when data are collected over laps
  - with these data, small changes in CdA are estimable
  - it's possible to examine how these estimates are affected by air density, wind speed, and wind direction
- knowing speed, power, and a little about the course provides even more information
  - you can tune the model not only to line up the profiles but also to match total elevation gain
- in some cases, knowing a bit more info can help you to get separate estimates of Crr and CdA (if that's what you need to do)

# main conclusion

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- up to now, if you had an on-bike power meter, most field test protocols required that you find a flat venue and perform tests at constant speed
- **Using this approach eases data collection and greatly expands the number of appropriate field test sites you can use**
  - you can get good diagnostics of when a test run didn't work so you can improve your experimental technique
  - model deviations are localized so you can spot them
  - precision is no worse and can be better than classic method
- wind is still a problem
  - but diagnostics provide a clue for when to worry and you can sometimes “crop” your data to exclude short interruptions (such as a passing car)

# what kinds of venues work?

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- flat road or velodrome, as for “classic” approach? *yes*
- industrial park loop? *yes*
- single out-and-back up a slight hill? *yes*
- twice up the same hill at different speeds and power? *yes*
- long residential block shaped like a “half-pipe” with a dip in the middle? *yes*
- almost any wind-sheltered route where you don't use brakes? *yes*  
    ideal venue could be a bowl-shaped route that lets you speed up and still slow down at the ends to make the turnaround
- city street with stop signs, wind gusts, and passing traffic? *nope*

# recommendations

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- use the “modified classic” method if your venue allows. If not, do laps or loops
  - shorter laps let you do more of them
- don't hold speed constant
  - the wider the spread across laps the easier it is to isolate separate effects
    - a small amount of elevation change can help increase speed variation as long as it's not so steep you need to brake
- measure air density, don't use your brakes if you can help it, and if you're trying to detect small changes do this on a wind-free day, or use an on-bike wind meter



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## estimating CdA with a power meter

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Cyclists interested in going faster want to reduce the amount of drag that they must overcome.  $C_d$  is the coefficient of aerodynamic drag.  $A$  is front surface area. Their product,  $C_dA$ , is also known as “drag area” and is typically measured in square meters.  $C_dA$  can be thought of as approximating the fraction of aerodynamic drag compared to a cube with front surface area  $A$  held perpendicular to the direction of motion. Sometimes you'll see  $C_xA$  instead of  $C_dA$  to specify drag in the direction of travel even if the wind is not from directly ahead.  $C_d$  usually ranges between 0 and 1, though direct measurement of  $C_d$  is difficult to do so we usually talk about the product,  $C_dA$  (as it turns out, directly measuring  $A$  isn't a piece of cake, either, though from a history of science perspective some measurements of  $A$  have demonstrated quite a bit of ingenuity) and it is measured indirectly by drag force. An average-sized cyclist may have a  $C_dA$  that ranges from around  $0.2 \text{ m}^2$  (in a very good aerodynamic time trial position) up to perhaps  $0.8 \text{ m}^2$  (on an upright “city” type bicycle).

The gold standard for measuring  $C_dA$  is the wind tunnel but wind tunnels appropriate for testing aero drag on a bike are relatively expensive (\$500 - \$1000/hr, plus travel costs). Since the demand for knowing  $C_dA$  (or, at a minimum, determining differences in  $C_dA$  between alternative positions or equipment) is high, riders have turned to field testing to try to estimate  $C_dA$ . Field testing introduces its own set of problems: notably, that at best it can measure total drag (=aero drag + rolling resistance drag) so the aero component must be separated out. Traditionally, rolldown or coastdown tests were used though high variability in the estimates has been observed. With the introduction of on-bike power meters results have been both more accurate and more precise—but finding the right venue and collecting the right data to achieve good estimates is still challenging.

This presentation looks at real-world data collected by an on-bike power meter to assess how well these data can estimate  $C_dA$ . In particular, it shows that the usual approach is not robust if data collection is less than perfectly controlled, presents an alternative indirect method that can produce good results under more general conditions, and uses these findings to make recommendations about how to minimize critical errors and to improve estimates.

## with good data, field testing works

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- a validated power model  
Martin, et al. (1998), “Validation of a mathematical model for road cycling power”, J App Biomech 14(3)
- examples with good data collected in field  
Martin, et al. (2006a), “Modeling sprint cycling using field-derived parameters and forward integration”, MSSE 38(3):592-597  
Martin, et al. (2006b), “Aerodynamic drag area of cyclists determined with field-based measures”, Sportsmedicine 10: 68-9  
Snyder, J.; and T. Schmidt (2004), “Determination of drag parameters utilizing a bicycle power meter”, HPeJ issue 1  
*... but what happens if the data aren't good?*

Sometimes we can directly observe and measure the object or quantity we're interested in. In cases where we can't, sometimes we can measure other related variables and calculate the thing we want indirectly by using known relationships among the variables. All field-based methods for estimating CdA are indirect. We'll discuss two: the “classic” regression method and one introduced here.

When carefully done, field testing can produce good results – so why would anyone care about using less-than-perfect data? The main reason is that sometimes, no matter how hard you try, the data you collect aren't perfect. We have three questions:

1. How lousy can field testing be and still produce reasonable results?
2. When faced with less-than-perfect data, are some methods better than others?
3. What can we learn from this that can help improve field test estimates?

I come from a field where, for the most part, we can't run experiments and the data we work with tends to be expensive and difficult to replicate. Because of this we've developed a toolbox of techniques to salvage “dirty” data using robust indirect methods. Although the usual goal is to salvage bad data, perhaps the most valuable lesson is helping us to improve data collection by learning which data elements are critical, how sensitive the final result will be to errors, and how to evaluate that sensitivity by developing a method to measure goodness-of-fit. That's important, so let's repeat it: finding a way to make an estimate is only half the job; the other half is finding a way to tell when the estimate is lousy, and by how much.

What I'm presenting here may seem like the long way 'round the problem but it's done this way for a reason: I'll show one way of many to make an estimate but, more importantly, how to construct diagnostics that you can use to tell when the estimate is lousy.

## the classic approach

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- constant speed runs on flat windless roads
  - some alternatives: coast down tests, velodrome runs
  - often, results averaged over runs taken in opposite directions
  - occasionally, a few other adjustments and variations
- for constant speed on flat windless roads, power-drag equation simplifies to
$$\text{watts} = k_0 v + k_1 v^3, \text{ or } \text{watts}/v = k_0 + k_1 v^2$$
- so regress drag force (i.e., watts/v) on  $v^2$ 
  - the regression intercept ( $k_0$ ) is related to  $C_{rr}$
  - the regression slope ( $k_1$ ) is related to  $C_dA$

Other approaches include John Tetz' coastdown method, occasionally used among HPV users. See <http://www.recumbents.com/mars/pages/proj/misc/coastdown.html>

In the Human Power eJournal, John Snyder and Theo Schmidt present a method similar to the classic method here: <http://www.hupi.org/HPeJ/0005/0005.htm>

H.W. Schreuder presents a high precision coastdown method in <http://www.xs4all.nl/~cp4trml/metingen/measurements.html> using a datalogger similar in spirit to the one described here: <http://www.hupi.org/HPeJ/0012/0012.html>

Candau used coastdowns in a hallway timing trap using electric "eyes" similar to the ones used to ring a bell when a customer walks into a shop. See Candau, R. et al. (1999) "Simplified deceleration method for assessment of resistive forces in cycling." MSSE 31(10): 1441.

Andy Coggan reports that classic "regression" field testing under ideal conditions performed by an experienced user can produce estimates with an sd of 0.0025 - 0.003  $\text{m}^2$ . If so, this suggests that under these ideal conditions differences in components or position around .005  $\text{m}^2$  can be reliably discerned. For comparison, this is roughly half the area of a 3"x5" index card.

## the challenge

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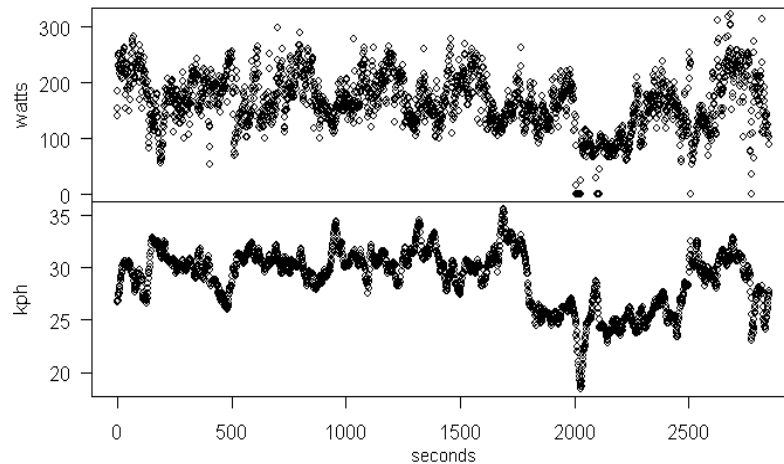
- I recorded power and speed during a ride consisting of a number of laps around a closed course
  - power was not constant
  - speed was not constant
  - the course was not flat
  - the wind was blowing (weakly but (I believe) consistently and from the same direction during the entire ride)
- how good of an estimate of CdA is it possible to get with these (lousy) data?
  - using usual approach, not good at all
  - using approach described here, not bad at all
  - with non-lousy data, you can get very good results

At first glance, it seems these data are neither controlled enough nor detailed enough to produce a reasonable estimate of CdA. In fact, even that first glance may be optimistic: unlike most cyclists who would attempt CdA field testing, I knew neither my exact weight nor that of my bicycle, nor did I measure wind speed or the elements I needed in order to calculate air density. Given these circumstances, most people would dismiss the estimation as unmanageable.

However, not all methods used for estimating parameters of a model are equal. Here, I describe a method that, under certain broad conditions, can be used to make a reasonable estimate of CdA from data such as these – in certain narrow situations, one that is accurate, consistent, and with high discrimination and repeatability. Of perhaps even greater importance, I present diagnostic procedures to recognize when those conditions don't apply so you won't be misled into thinking the estimate is accurate or consistent when it isn't. I will also discuss some of the weaknesses of this approach and will show that the challenge data reveal characteristics that allow a reasonable estimate even though all we know are speed, power, and that the course consisted of a series of laps. I conclude with recommendations for field testing of aerodynamic drag.

## the data

data were collected at 1.26-second intervals with a Power Tap hub.  
The plot shows that neither power nor speed were constant



You can download the data and experiment with them yourself:

<http://anonymous.coward.free.fr/wattage/cda/field-cda-challenge.csv>

The first few lines of the data file look like this:

```
"secs", "watts", "kph"  
1.26,142,26.8  
2.52,154,26.7  
3.78,185,26.7  
5.04,253,26.8  
6.3,247,27.5
```

i.e. all we know are speed, power, the time interval, and that the data are in sequential order.

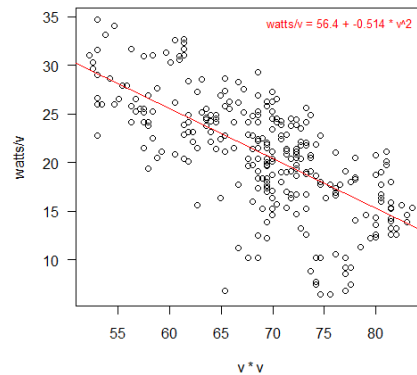
Torque was zeroed before the ride according to manufacturer's recommendation. The hub was checked with a static calibration test against known weights about 3 months before the data were collected and found to be (well) within specified limits for accuracy. That is, these data were collected with a standard hub in ordinary working condition using nothing special that isn't available to anyone who has a Power Tap hub.

## the problem

- flat, windless venues are hard to find (some use airplane hangars, building hallways, stadium concourses)
- the regression approach is not robust to changing speed, power, or conditions

using a 5 minute subset of the data produces a regression slope that is *negative* (and highly statistically significantly so), meaning negative CdA

**usual methods don't work well with these data at all**



Remember, CdA is an *area*. You can't have negative area.

In a deeper sense, we started off knowing that these data didn't fit the standard assumptions of constant speed on a flat course, so we shouldn't be surprised that the fit isn't good.

However, this shows that the fit isn't just not good, it's *catastrophically terrible* – and that in this context the usual regression approach isn't at all robust to failures of the assumptions. Part of the reason why regression has become such a popular technique is that when it fails it often fails gracefully. Alas, we've just demonstrated that “often” does not mean “always,” and explains why testers try to control the conditions as carefully as possible – yet often they still don't get good estimates of CdA.

Why is this non-robust approach the usual one? I suspect, but do not know, that the answer is historical: before the introduction of devices that could record speed and power, analyzing detailed moment-by-moment data simply wasn't an option. The usual approach works well when all one can analyze are averages collected over flat runs at constant speed; at that, it was an improvement over coast-down or roll-down tests. Candau et al. (1999 MSSE 31(10):1441-7) used coast-downs in building hallways to control slope and crosswinds. Jim Papadopoulos, who helped revise the latest edition of *Bicycling Science*, suggests testing on flat running tracks with an extra cycling computer set to show average speed so the rider can control speed even more strictly. In addition, performing regressions over a series of averages is relatively easy so at a time when data were sparse, computers were rare, and speed and power recorders were nonexistent, it was an approach that simplified data analysis.

You may be able to discern a philosophical conflict here: the experimental approach is to tightly control conditions which simplifies the analysis. I come from a field where we can't run controlled experiments so we develop (slightly?) complex analytical tools, then ask what happens if the conditions were poorly controlled.

## a different approach

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- record moment-by-moment speed and power from a series of laps on the same route
  - route need not be flat (in fact, it's better if there's some change in elevation)
  - speed and power need not be constant (better to vary speed and power)
  - hold position and don't use brakes
  - wind should be as close to zero as possible
- construct an elevation profile for the ride as a function of known power, speed, mass, and air density, and initial guesses at CdA and Crr. Plot the elevation profile against distance
- since each lap must start and end at the same place, find the value of CdA that produces zero net elevation gain over each lap. One (but not the only) way to do this is to try different values until the laps “line up”

The third point is key. Nothing comes for free and this approach adds an additional constraint that is absent in the conventional approach: elevation gain must net to zero for each lap. (Later, I'll show an approach that lets you use a course with a known true elevation gain.)

When I first tried this approach in 2003, it was prompted by an attempt to examine how well the HAC4 could estimate power from elevation change. A side effort was to “reverse-HAC” power data to see how well I could back out elevation change. I described it at

<http://anonymous.coward.free.fr/wattage/altimeter/pseudohac4.html> and

<http://anonymous.coward.free.fr/wattage/altimeter/altimeter.html>

In contrast to the usual approach, this approach demands much more data and a bit more calculation—but requires much less control over the conditions of the test. In essence, it depends on hundreds of times as many observations (in known sequential order) to correct for deviations from controlled conditions: it clearly would have been impractical to do before the advent of modern power meters and analytical tools.

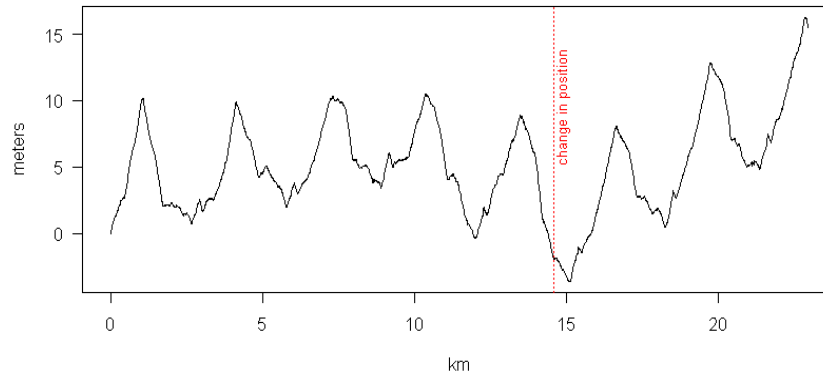
Note that coast down tests are a subset of these conditions: the route isn't flat, speed isn't constant, you must hold position, don't use the brakes – but power is constant at zero.

There is another irony: although in my real job I usually teach statistical research methods, this particular approach doesn't use standard statistical techniques as does the conventional approach. Here, we exploit physics modeling to estimate one of the model parameters on a moment-by-moment basis, then examine the behavior of that parameter. There are several model parameters we could have modeled but here we choose to model slope. The virtue of this particular parameter is that it lets us evaluate the course profile as an overall diagnostic of fit. We then tune the course profile so it matches from lap to lap.

There are ways to assess fit and precision more formally and we'll discuss those a bit later.

## Q: did we correctly identify laps?

this approach provides a self-check: it should identify the correct number of laps. Analyzing these data reveal seven-and-a-half laps with about 10 meters of elevation change per lap. Was that right?



There are two things I didn't mention in the original challenge, and they both emphasize why I begin by describing this method as a graphical tuning: first, it appears that there may have been an extra half lap; and second, it's easy to see that the first few laps lined up, while the last couple of laps seem like they were collected under different conditions. Both of those turn out to be true: I entered the course in a different spot than I exited, and I sat up for the last couple of laps. The dotted red line shows when I changed position from brake hoods to bar tops. On my bike, I have the brake hoods in “classic” position (i.e., lower than is now popular) so there is a height difference between being on the hoods and being on the tops. On the other hand, when I'm on the tops, I usually hold my hands nearer the stem, so I'm narrower. I did not expect that the difference between being lower but wider vs. higher but narrower would be so easily spotted.

The take-home message is that while it's possible to do this analytically (and I will, later in the presentation) it's important to start graphically because the graph is a key diagnostic tool.

Note the scale on the x-axis is in km and the scale on the y-axis is in meters – the vertical scale is exaggerated by two orders of magnitude. This is a flat course.

A final observation: although the course was closed to cars, there were other riders on the course. I was able to do the entire ride shown without using my brakes except for one brief moment. Notice the flattened top of the third peak? That's where a rider in front of me went wide and I feathered my front brake to avoid him. The other peaks are much more pointed. Does it make sense to you that a brief use of the brakes should appear as it does?



## A: pretty much, yes

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using *only* speed and power, we identify key features of the ride

correct number of laps? *yes*

correct lap length? *yes* (3.12km)

entered on one side of course and exited on other? *yes*

entered at “bottom” of course and exited at “top”? *yes*

10 meter elevation change over each lap? *close* – I believe it's closer to 15 or 16 meters

correctly identified high and low points within laps? *yes*

shows conditions were not constant (i.e., change of position during last two-and-a-half laps)? *yes*

brief use of brakes on third lap? *yes*

This method identifies features of the laps quite clearly from just power and speed.

The method missed on elevation, but notice that it didn't show random fluctuations over the course, or 200 meters of elevation change: it correctly identifies the course as nearly flat, with the same features in the same places on each lap. Remember that we've assumed the wind was zero although we know it wasn't. This method tosses all unmodeled variations into the estimated slope so the wind shows up as “virtual” elevation and gives us a rough idea of how strong the wind was: it translated into about 5 fewer meters of elevation change for each 3.12 km lap.

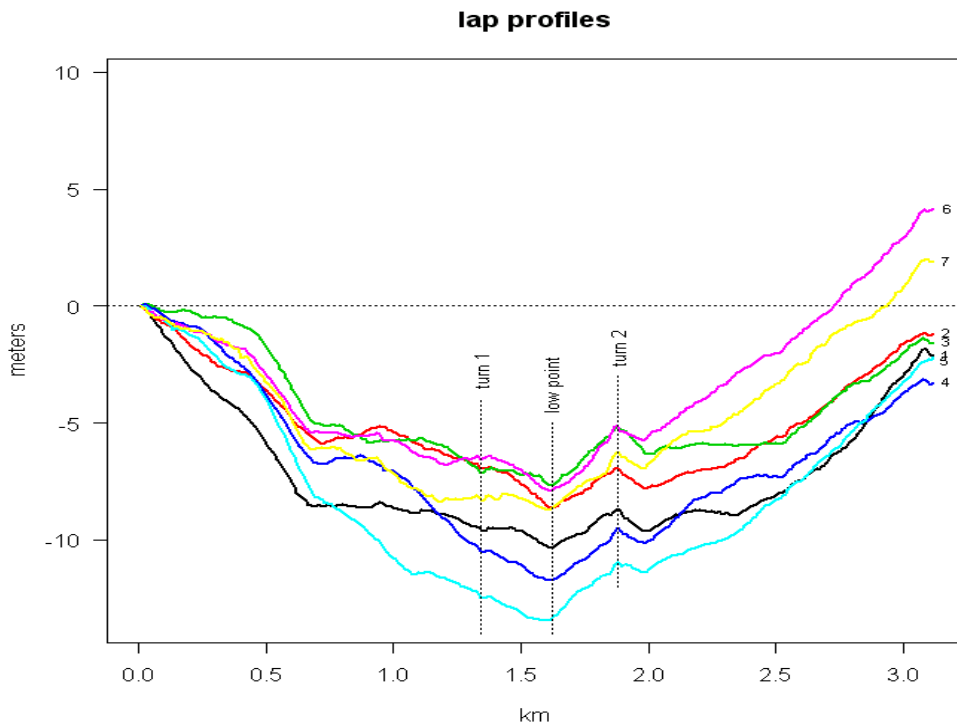
This is a clue as to why it doesn't matter that much what my total mass is. This is a relatively flat course, so we'd expect that total drag force would be dominated by aero drag, not drag due to lugging my fat rear up a hill. As we will later see when we explore the sensitivity of the method, a 1% increase in total mass resulted in a 0.3% decrease in estimated CdA, i.e., I could be off by 7% on my weight and the error in the estimated CdA would only be about 2%. On this course the estimate of CdA is pretty inelastic with respect to mass. Keep this in mind when we discuss hills.

We now know the lap length to be very close to 3.12 km -- we'll use this information later. (How close? The PT records at 1.26 second intervals so between adjacent records I could travel 8 to 10 meters). In the meantime, note that the wind doesn't appear to change the lap length very much in any of the data files we'll be looking at. If the wind is gusty and strong (relative to the actual gradient) and if it changes direction then the lap lengths may be harder to identify. This is a foreshadowing of a useful diagnostic that we'll discuss later.

## lap lengths are well identified

- different guesses about CdA (or Crr or mass or air density) have only a small effect on the estimate of lap lengths
  - changes in the parameters move the curve up-and-down but not left-and-right
- small changes in wind don't affect lap length much so lap lengths are relatively robust
  - however, big changes in wind may
- relatively robust identification of the lap lengths means that it's feasible (though not always wise) to impose the “zero net elevation gain” constraint on each lap
- if you're interested, a Google map of the course is here:  
<http://tinyurl.com/yq9r76>

The route follows a clockwise course: northward up the Route Dauphine, eastward onto Route Royale de Beaute, southwest down the Route de Bourdon, and finishes with a hairpin back onto Dauphine. For these data, I started at the northeast corner and exited at the hairpin. According to Google Earth, the high point is at the hairpin, at 62m ASL; the low point is around the “e” in “Royale” at 46m. This surprised me—my “feeling” from riding the course was that the ride was a bit flatter, and that the low point was on Dauphine. As an aside, if you flip to the satellite view and zoom all the way in, you can see individual riders.



## why plot?

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- I could have done (and will later show) this algebraically
- however, for now it's easier and perhaps more instructive to plot graphs and show what's happening

algebraic solutions generally look for a parameter that maximizes some overall measure of fit

in this case, you learn more by looking for *specific* areas of fit and misfit rather than *overall* fit

the graphical approach makes it easy to find lap length and knowing lap length will be useful

*perhaps most importantly*, the plots give us a generalizable way to diagnose lousy estimates

One closed-form algebraic approach is to transform the power equation into a work equation, then solve for the CdA. To do this, integrate the power equation over time. I'll come back to this later.

Diagnosing lousy estimates is key. Lots of methods give you estimates but either do not make it easy to evaluate how good those estimates are or else base their evaluation on models of random sampling. Here, we've converted puffs of wind, or the use of brakes, or a change in position, into a common metric, viz., elevation change, which is easy to visualize. For example, a brief use of the brakes can be spotted as a sudden and very localized jump of half a meter or so in elevation. A car passing you from behind can often be distinguished from a car passing in the opposite direction. The advantage of this approach is that the true elevation doesn't change from lap to lap so localized disturbances are clear and obvious. This is not true for something like “virtual wind” since in the field we can't guarantee that the wind is the same from lap to lap. If you're trying to check the calibration of your power meter it's better to check with weights of known fixed mass than it is to wobble your body weight on the pedal. Likewise, it's much easier to spot deviations from a model when you're comparing against something fixed (like fixed weights or elevation) than when you compare against something that is variable (like a wobbly person or wind).

## so what was the CdA?

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- hold your horses. First we have to check the assumptions and calculations. To do that, you need to know how to do them.
- we'll start from the beginning, with the power-drag equation, and split the analysis into two parts:
  - assuming no wind
  - assuming some wind, but wind which is consistent in speed and direction

For these data, we don't know wind,  $C_{rr}$ , air density, or total mass – though if you were going to collect your own data you should probably weigh yourself and check the weather service for barometric pressure, temperature, and your elevation above sea level (barometric pressures are often, but not always, normalized to sea level so you should double check for your location). I wasn't kidding when I said that these data were less than ideal.

Before we get to the estimate of CdA, we need to double-check the assumptions and how sensitive the method is to them.

## a (simplified) power equation

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$w$  = watts needed to propel bike at speed  $v$

= watts to account for rolling resistance +  
watts to account for change in elevation +  
watts to account for change in speed +  
watts to account for air resistance

$$= w_{rr} + w_{PE} + w_{KE} + w_{aero}$$

Many models include a term for overall drivetrain efficiency,  $\eta$ , but all of the data files I'm looking at come from Power Taps which, in theory, should be downstream of drivetrain losses, i.e.,  $\eta = 1$ . If you have an SRM, which measures power at the crank (i.e., upstream of drivetrain losses), you will want to decide how to model drivetrain losses. Martin et al. presumed a fixed percentage loss of 2.3% of power (i.e.,  $\eta = 0.977$ ). Other choices might include a fixed wattage loss, or loss with two components: a fixed amount and a fixed percentage.

PE is “potential energy” and represents change in elevation.

KE is “kinetic energy” and represents change in speed.

## simplified power equation, continued

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$$\begin{aligned}
 w &= W_{rr} + W_{PE} + W_{KE} + W_{aero} \\
 &= C_{rr} m g v + s m g v + m a v + \frac{1}{2} C_d A \rho v_{air}^2 v
 \end{aligned}$$

where

$v$  = speed in m/s (i.e., “ground” speed)

$m$  = total mass (kg) of rider + bike

$g$  = 9.81 m/sec<sup>2</sup>

$C_{rr}$  = coefficient of rolling resistance

$s$  = slope

$a$  = acceleration

$\rho$  = air density

$v_{air}$  = “air” speed of bike

$C_d A$  = drag area

When there's no wind, ground speed=air speed. Then, if PE and KE =0, (i.e., if the course is flat and speed is constant) you can get regression approach.

One of the reasons this is a “simplified” model is because it ignores yaw angle, i.e., the angle at which the wind strikes the rider.  $C_d A$  varies with yaw angle but if wind is low relative to rider speed, the yaw angle approaches zero. If there is no wind at all, the yaw angle is exactly zero.

Typically, you'd know your total mass and you'd record temperature, altitude, and humidity in order to calculate air density,  $\rho$ , so one way to solve the equation is to make an initial guess at either  $C_{rr}$  or  $C_d A$  and calculate the other.

Although it appears that we are assuming fixed  $C_{rr}$ , a more precise statement is that we only need to assume that whatever changes in  $C_{rr}$  occur, they do not change from lap to lap, e.g., the lap can have rough patches and smooth patches but they don't migrate randomly around the lap. We're assuming that each time you ride over the same spot the  $C_{rr}$  in that spot will be the same as it was on previous laps.

Earlier I mentioned an algebraic approach. Integrate the power equation to convert it to units of work in joules. Then total work for *any* segment of a course is:

$$J = J_{rr} + J_{PE} + J_{KE} + J_{aero}$$

That's true for any segment. In particular it's true for specific segments of a course that correspond to laps, so the net elevation change is zero. Reformulating the power equation into a work equation lets us factor out  $C_{rr}$  and  $C_d A$ . Then, given a particular value of  $C_{rr}$ , we can solve for  $C_d A$ .

If we knew from external information what the actual change in elevation over a segment was, we could use that information to make estimates over that segment rather than whole laps. For example, suppose part of your lap included a hill for which you knew the altitudes at the bottom and the top. You could use this method on that segment alone.

## no wind approach

- assume  $v_{\text{air}} = v$  and solve for slope as a function of other variables

$$s = w/(m g v) - C_{rr} - a/g - (\rho C_d A v^2)/(2 m g)$$

- use this formula to estimate point-by-point slopes from the data, supplemented by initial guesses at  $C_{rr}$  and  $C_d A$ . Ballpark guesses for starting values might be  $C_{rr} = .005$  and  $C_d A = 0.3$ .  $v$  is in meters per second, so convert  $v = \text{kph}/3.6$ . Since these data were collected at 1.26-second intervals a reasonable estimator for  $a(t)$ , the acceleration at time  $t$ , is  $(v(t+1.26) - v(t-1.26))/(2 * 1.26)$

standard approach assumes accelerations = 0. This approach calculates and uses them

- use estimated slopes to construct elevation change for each 1.26-second interval:  $\text{elev.change} \approx s * v * 1.26$

We'll assume wind speed is zero right now, but we'll see what effect it has later.

There are other ways to calculate the accelerations but this method is relatively simple and reasonably close—since the power and speed being reported are the average power and average speed over the preceding interval, the “simple” estimator of  $a(t) = (v(t) - v(t-1.26))/1.26$  lags behind the “true” acceleration by about half an interval. The approach here corrects this in two parts: first, you need to find the “instantaneous” speed at time  $t$ , which could be something like  $v^*(t) = (v(t) + v(t+\Delta t))/2$ ; second, estimate the acceleration appropriate to the interval  $t$  as the difference in the instantaneous speeds, or  $a(t) = (v^*(t) - v^*(t-\Delta t))/\Delta t$ , which simplifies to  $(v(t+\Delta t) - v(t-\Delta t))/(2\Delta t)$ ; in our case  $\Delta t = 1.26$  seconds but you might be collecting data with your power meter at a different time interval. You'll also need to make “endpoint” adjustments for the first and last intervals, but these usually have a pretty minor effect.. The exposure corrected acceleration then has the virtue that solving for the “flattening”  $C_d A$  analytically in work domain (i.e., in Joules) preserves the right VE profile.

$\text{elev.change}$  ought to be  $\sin(\text{atan}(\text{slope}) * v * 1.26)$  rather than  $s * v * 1.26$  but I usually ignore the  $\sin(\text{atan}(\text{slope}))$  correction: the difference usually is no greater than a millimeter or so and the errors in the reported values from a power meter can be greater than that. For example, most power meters report nominal speed only to the nearest 0.1 km/h (some are worse than that) so the trigonometric correction is probably overkill. However, there's no harm in including it if you want.

You'll generally know your mass and be conscientious enough to take the measurements you need to calculate  $\rho$  but for me on that day  $m = 84\text{kg}$  and  $\rho = 1.2$  are close enough. You'll see later that for these data the estimate is relatively insensitive to  $m$ . Later, we'll re-examine the “relative” insensitivity to changes in mass.

## produce an “elevation” profile

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- cumulate the “elevation” changes and plot against distance to produce a “virtual elevation” profile
- later we'll see how to solve algebraically for the CdA that makes the plotted laps line up but for now just make guesses at CdA to achieve that
  - the advantage of making iterative guesses at the CdA and re-plotting is that you can see whether the elevation profiles are well-behaved and learn how sensitive or robust the CdA estimates are
- these steps may seem daunting but they only take a few commands in any spreadsheet or programming environment like R or Matlab, and have already been implemented in Golden Cheetah, the open-source cycling analysis program

I put “elevation” in quotes because what we're really calculating is a “virtual” elevation. It's “virtual” because anything that is left unmodeled (like wind or using your brakes) is tossed into the slope, and we're using slope to calculate the elevation profile.

To get distance (in km) from speed (in m/s), cumulate  $v * 1.26 / 1000$

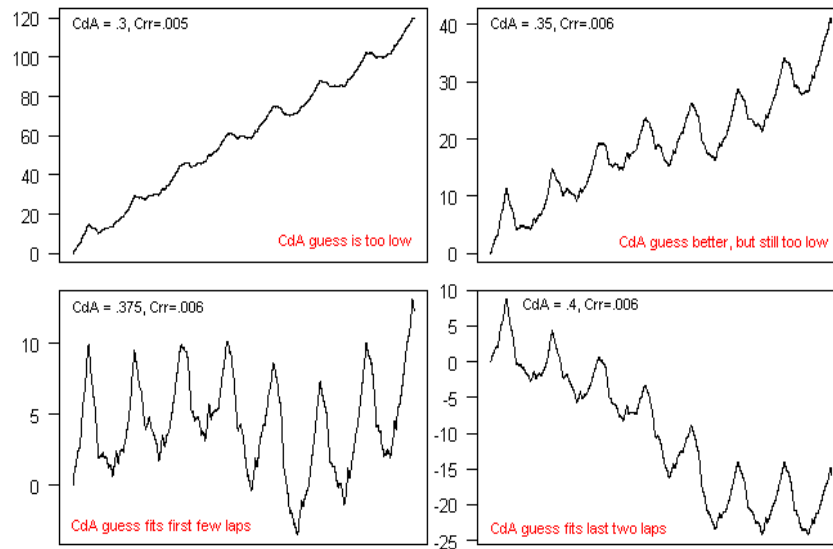
You may recognize that what we're doing is getting an elevation profile for the ride by integration of estimated elevation change over distance.

Note that this cumulative approach uses the raw and unsmoothed point-by-point slopes. Because of the limited resolution of the data (that is, the data are collected at 1.26-second intervals, and speed is recorded in tenths of km/h), you may think it's good to slightly smooth the calculated elevations. I have not bothered to do this because the calculated profiles are already smooth enough, and cumulating acts as a built-in smoother (you may recall from integral calculus that the integral of most functions is usually smoother than the function itself; and that the derivative of a function is usually rougher than the function itself).

Although I have implemented this method in a set of R functions, not everyone uses R. Others have created spreadsheets to do (some) of the analysis and they are far more accessible to most readers.. Here's a link to a spreadsheet created by Alex Simmons:  
[http://wattage.googlegroups.com/web/AeroTestVirtualElevation\(Chung\)Method.zip](http://wattage.googlegroups.com/web/AeroTestVirtualElevation(Chung)Method.zip)



## estimated CdA should level the profile



The upper left panel shows that the profiles creep upward, a clear symptom that the putative estimate of CdA is too low. Also note the y-axis vertical scale: it appears that over the course of what will soon be recognizable as seven-and-a-half laps, the total cumulative elevation gain was about 120 meters, or about 16 meters of net gain per lap. Nonetheless, you can already see the rough outlines of laps.

The upper right panel uses a higher estimate of CdA, and also of Crr (of which I will later say more). Now the number of laps is clear. The vertical scale has shrunk, indicating perhaps 5 meters of net gain per lap. Still, we need to increase the estimated CdA.

The lower left panel shows what appears to be a stationary pattern for the first three laps, i.e., zero net elevation gain over those laps. There appears to be a slight fall in lap amplitude, then a secular upward drift. This is a symptom that the CdA changed from the first few laps to the last couple of laps. As explained earlier, even though I did not mention it when I first presented the data, I sat more upright for the last couple of laps—and I also had to lightly hit the brakes at one point in the ride. The method picks up both of these changes. This panel is the same image as the larger version you saw earlier.

The lower right panel raises the CdA once more, to a level where the last couple of laps appear stationary. This identifies a combination of CdA and Crr that could apply over the last part of the ride.

## why does this work?

---

classic approach regresses average drag force on  $(\text{avg } v)^2$  and minimizes sum of squared errors. Instead, we minimize the sum of a more complex form of the error: we integrate the moment-by-moment elevation changes across distance, then impose a new constraint that the elevation gain across laps must net to zero

laps are extra information that the classic approach ignores. In addition, the data are sequential. Sequencing the data means accelerations can be calculated and included instead of assuming they are zero. There are other ways to produce a “solution system” but the elevation profile is a convenient way to maintain the sequence of the data, to allow for the additional constraint on fit, and to provide a diagnostic of specific areas of fit and misfit

Minimizing squared error has certain very desirable properties from the point of view of statistical inference; however, if the method is not robust to error, statistical inference is unimportant.

In spirit, this is similar to the method of maximum likelihood; in this case, CdA is chosen to maximize the “likelihood” (loosely defined) of observing elevation profiles with zero net elevation gain from lap to lap. Another approach that may be familiar is Laplace transforms. Like Laplace transforms, this is an integral transform that converts a sequence of data collected in time domain into a function of a different variable: in this case, distance and elevation as a function of CdA. Of course, a transform only makes sense if the alternative either simplifies the analysis or provides some new insight on the relationships in the original form. That insight is the elevation profile implied by the data. In addition, we're adding laps so we can create a new constraint and exploit a natural “periodicity” in the data. That kind of periodicity doesn't exist in other approaches. Later we'll see other ways to create the proper periodic contrasts in different settings and that will give us hints on how to construct good test protocols.

As mentioned earlier, another way of thinking of this is as a generalization of a coast down test. In a typical coast down, you coast from a known speed down to another known speed on a surface of known slope. In that case, you're applying a known power: zero. In this case, you're doing a “coast down” with known non-zero power, and using the recorded speed to tell you how quickly you're decelerating. See H.W. Schreuder (op. cit.) for a discussion of high precision coast downs.

## when doesn't this work?

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- this method models moment-by-moment power as a function of moment-by-moment speed and changes in speed but *everything else* gets tossed into a residual that we then model as if it were a slope. That's why what we get is a “virtual” elevation profile
- if 1) there are errors in measurement, or 2) the unmodeled parts of the power equation (like wind or brake usage) are large relative to the modeled parts, or 3) CdA changes because you didn't hold your position, or 4) you're losing air in your tires so the Crr is changing systematically, then the virtual elevation profile will differ from the true elevation profile

we'll see more discussion of this when we talk about wind

Note that the flatter the course the greater potential effect of unmeasured wind since wind will be larger relative to the true elevation.

You want the errors to be small relative to the modeled parts. In the usual approach you tightly control speed, acceleration, and the slope and you choose windless days. In this approach you don't have to control the speed and acceleration since they're measured well. However, you want a good spread of speeds and a reasonable amount of change in elevation to help “isolate” wind effects.

That is, if you know the true elevation profile it gives you a good way to assess how much the estimate was affected by unmeasured wind. This turns out to be useful: the usual approach is to wait for a wind-free day, to test on a flat (or constant slope) road, to hold speed constant (or, at least, to minimize changes in speed) but there is no simple way to tell if the measurements were tainted by wind, or changes in speed, or a small degree of slope, or a slight change in position.

If you ride laps, you can “overlay” them to see how similar the VE profiles are for each of the laps. If they're very different, you know it was too windy, or you didn't hold your position, or something else happened to the measurements, and you can see if the difference was transient or perduring.

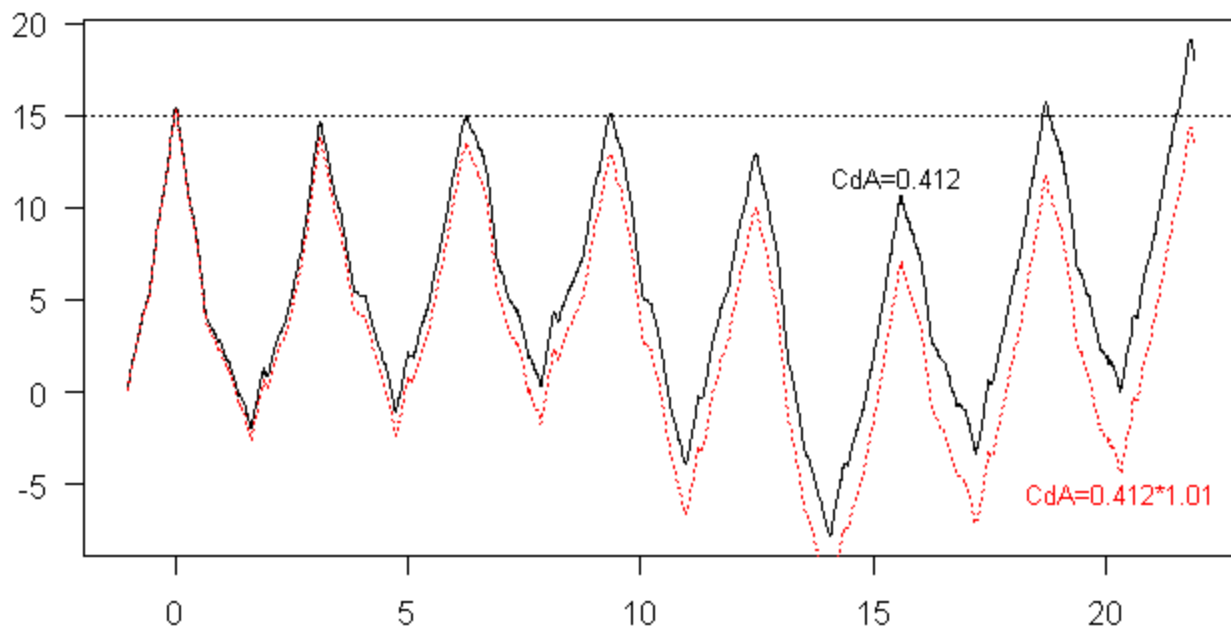
Here's another way to think of it: we're trying to raise the “signal-to-noise” ratio. The classical approach to field testing tries to increase this ratio by decreasing the noise. Decreasing noise is always a good thing but another approach is to decrease noise and to increase the signal. This approach models accelerations and “sequences” the data in order to increase the signal, then re-casts the model in a way that lets us measure deviations from fit.

## absolute and relative CdA

- in this example I made guesses about both CdA and Crr. Look at the equation—Crr and slope have the same “scale” so an increase of .001 in Crr looks like an increase in the slope of .001 (=0.1%). Later this will turn out to be handy.
- Nonetheless, *with these data*, we appear to have pretty good relative accuracy but unless we know what Crr is, we won't have good absolute accuracy
  - good relative accuracy means we can spot small *changes* in CdA even if (with these data) we can't nail down CdA itself. Sometimes you'll want do specific additional tests that will let you nail down both CdA and Crr
- I was bad and didn't measure air density (though I have a ballpark idea about what it was)

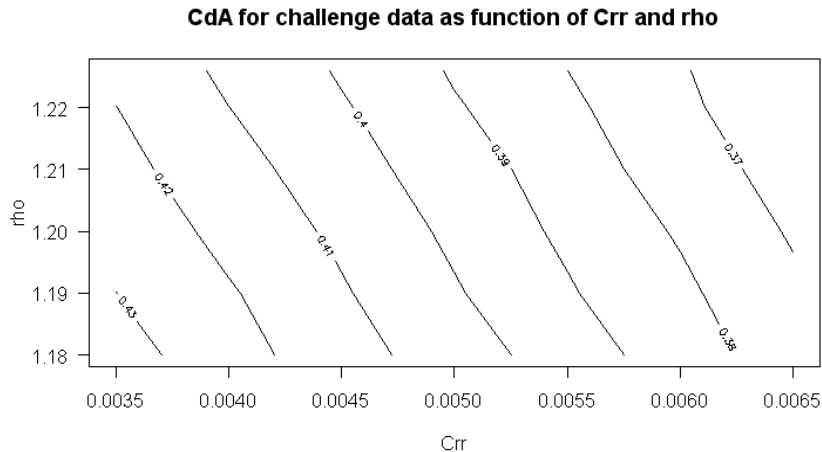
How small of a change can we spot? With these data, you can easily spot a change in estimated CdA of 1%. Using this method, one rider reported repeated estimates of CdA within  $.001 \text{ m}^2$ ; another added a 5cm x 5cm cardboard square ( $=.0025 \text{ m}^2$ ) to his bike and reported an estimated change in drag area of  $.003 \text{ m}^2$ .

### 1% change in CdA is discernible



## so what was CdA?

since there were so many things I didn't record, the best we can do *with these data* is to calculate CdA using a rough guess at my total mass and assuming different values of Crr and air density. We get:



Let's review: using only power and speed, we can show that the calculated profiles are relatively inelastic to mass. Most people doing field testing would at least make an attempt to measure air density but I didn't so the best we can do here is to produce an estimate for CdA that depends on Crr and air density.

For given Crr, increasing air density implies decreasing CdA, and a 1% change in air density implies around a 1.5% change in CdA.

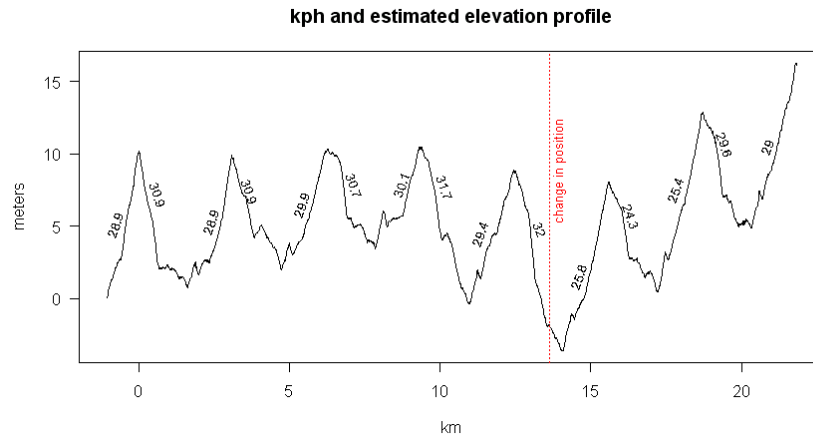
For given air density, increasing Crr implies decreasing CdA, and a 1% change in Crr implies around a 0.3% change in CdA.

This may make it sound like Crr is less important than air density, but air density is easy to measure and it changes relatively slowly while Crr is hard to measure well, road surfaces can change quickly, and changing road surfaces can change Crr by much more than 1%. The bottom line is that although CdA is relatively less sensitive to changes in Crr than to change in air density, the magnitude of changes in Crr can be large so the overall effect is also large. Conversely, if you're off on air density by a little bit, it won't affect CdA that much. Bottom line, you should probably do your best to record air temperature and barometric pressure, but don't sweat too much about air density changing over the course of your runs.

Here's an important observation: for these data, the total elevation change doesn't appear to be that sensitive to changes in Crr. That's so for these data but it will turn out that this is not always the case; in fact, we'll exploit this difference later.

## what about lap amplitude?

you may have noticed that the “amplitude” of the estimated elevation differed across laps. Could it be related to speed?



Being able to recover the elevation profile from the data lets us look at average speed over equivalent segments of the ride. Here I've split the ride into “downhill” and “uphill” portions and looked at the average speed over each segment. Note that I don't use the estimated  $C_dA$  and  $C_{rr}$  in this part of the calculation—we don't need them in order to identify lap lengths or the relative high and low points. Since we can identify matching parts of the laps by distance, we could have split the laps into arbitrarily many segments; I chose two for simplicity's sake.

## ground speed and air speed

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- there is rough evidence that for these data the elevation profiles are speed-dependent
  - increased speed in the downhill direction increased elevation change
  - increased speed in uphill direction decreased elevation change
- could it be unmeasured wind?
  - up to this point, we've assumed no wind (i.e., ground speed = air speed)
    - recall that the challenge included the information that there was an unmeasured amount of wind, but that I thought it was light and from a consistent direction

The profile we've estimated up to this point showed about a 10 m change in elevation over a distance of about 1.6 km, or an average slope of around 0.6%, so the “downhill” is not very downhill at all. However, I've also said I think the “true” elevation change is closer to 15 or 16 m, so I suspect the actual average slope is close to 1%. In fact, I believe the true slope is not even and there are slightly steeper and slightly shallower segments, with one short downhill segment close to -2%.

## a handy diagnostic

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now you can see why we started with a graphical approach even though we could have solved algebraically for an exact solution: the graphical approach provides a handy diagnostic for whether the model assumptions are met

unmeasured variables affect the profiles in recognizable ways

unmeasured wind typically makes the profiles speed dependent

unmeasured braking typically appears as a sudden jump in the estimated elevation

for an out-and-back the outbound and inbound profiles won't match

Just because I often start with graphical analyses doesn't mean that's all I do. However, graphical approaches are often a good way to summarize models and are especially useful when they can tell you about model failures and poor fit.

Often on an out-and-back time trial on a straight course the only time you use the brakes is at the turnaround. The turnaround will be obvious in the virtual elevation profile because there is a sudden jump in the profile at that spot. Sometimes it's possible to “cut-and-splice”: cut out a little bit of the file surrounding the turnaround and splice the two half-profiles together.

And now you also can see why I started with this example: the wind was strong enough to affect the estimated CdA but this method makes it easy to see that this happened.



## what about the wind?

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is it possible to say anything about the wind from the data we have?

we'll try adding a (small) non-zero tailwind for the downhill direction and an equivalent headwind for the uphill; then switch

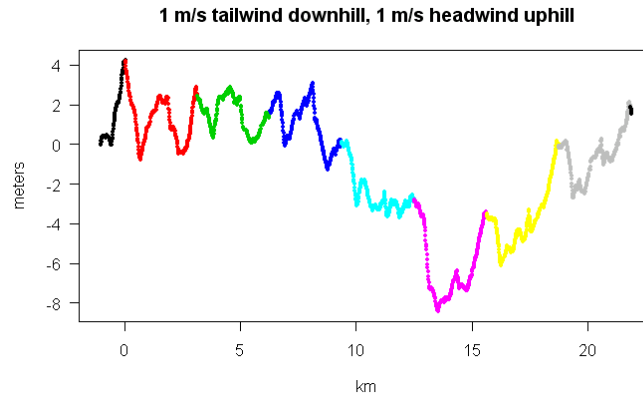
note that this is only a rough correction: the actual course was not a straight out-and-back so adding a small amount of tailwind and headwind is a simplification – the actual course was closer to a right triangle. In addition, roughly half of each of the “downhill” and “uphill” sides were exposed to the wind while the other halves were moderately protected

As previously mentioned, we could have split the laps into arbitrarily many segments, each with their own wind speed and direction. There is no particularly need to balance the wind in the downhill and uphill sections but we'll begin with this (obviously) simplified model. I gave a URL to the Google map for the course earlier.

## downhill tailwind, 1 m/s

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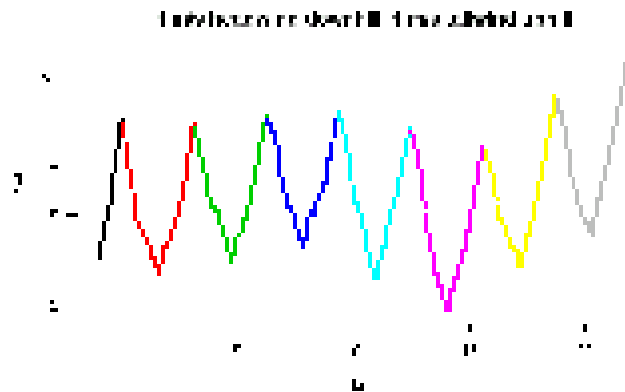
here's a new estimated profile, assuming a consistent 1 m/s tailwind in the downhill segment and a 1 m/s headwind in the uphill segment. Notice that the laps don't have the same shape



I've used color so you can see the beginning and end of each lap. Notice that the total elevation gain across the first few laps is around 4 meters.

## downhill headwind, 1 m/s

the lap amplitudes and profiles are much closer, and total elevation gain over each lap appears to be around 16 meters



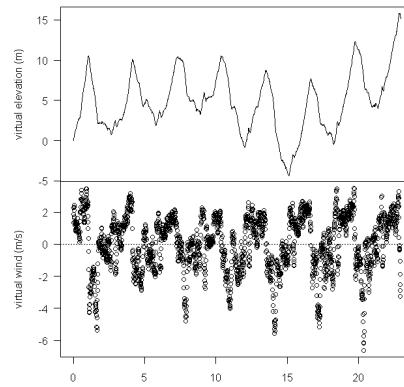
Even with this extremely simplified model, the total elevation gained across each lap is almost exactly what was shown on the topo: 15 to 16 meters from high point to low.

Doubling the windspeed to 2 m/s increases the elevation gain but doesn't change the overall shape (much).

## what about virtual wind?

- virtual elevation assumed zero wind. For virtual wind assume an elevation profile and see what the wind must have been
- can you see change in position for last two-and-a-half laps in the virtual wind plot, or the slight use of brakes at “top” of lap 3? (neither can I).
- virtual wind is much noisier than virtual elevation

that's why we model on VE rather than VW



For the virtual elevation calculation we assumed a particular “wind profile” (in this case, zero wind but if we knew more about the wind from, say, an on-bike wind sensor we could use it) and figured out what the slope must have been. For the bottom panel, we did exactly the opposite: we assumed a known elevation profile (in this case, that the course was absolutely flat but, just as above, if we had reliable elevation data we could use it) and figured out what the wind should have been. Of course, we know that the wind did blow and the course wasn't flat so our “true” profile and wind should be somewhere between the two.

If you plot virtual slope rather than virtual elevation (we calculated virtual slope in order to get virtual elevation), you'll see it looks very similar to virtual wind since in this example we assumed either zero wind or zero elevation change. However, even if we did include some wind or elevation data, the calculated slopes and/or wind will still be noisy. Integrating slope over distance to get the elevation profile is the key – and integrating wind over distance doesn't have the same easy interpretation because while we know that once we get back to the start of a lap the elevation nets to zero, no similar constraint applies to the wind.

In addition, we don't usually have the detailed wind speed data we would need in order to model on wind (though recent work on an on-bike wind meter may change that). That is, it's easier to check that a virtual bump or dip corresponds to a “real” feature of the road (we just check to see if it consistently appears in the same place) than it is to see that a virtual gust of wind was real rather than a misspecification of the model. I have examined a series of alternative virtual wind-virtual elevation scenarios and can unequivocally state that it ain't easy: It's sometimes possible to find a “putative” elevation profile that makes the virtual wind profile look not too different across laps but, unlike elevation, we don't usually know that the wind is constant.

## will it work with other examples?

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- would I be showing these to you if I thought it didn't?
- when Crr is known this method matches wind tunnel and classic field tests to within +/- 1%
- the following three examples illustrate the method with data not collected by me

three laps at Fiesta Island under windy conditions

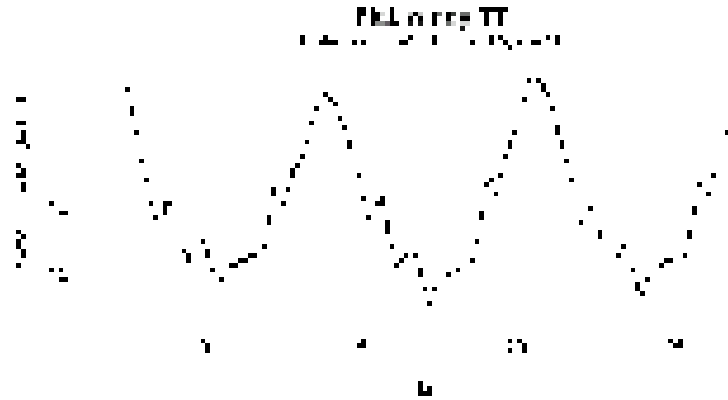
a (flat, windless) race on the track

Dede Demet's Montreal World Cup win: a hilly road race

Before we move on to the other examples, here's a little bit of information about the conditions under which the challenge data were collected: I don't know the exact total mass but 84 or 85 kg is probably pretty close. The temperature was in the upper 50's or low 60's (F, or around 15 to 17 C), it rained that evening, and you know from the description of the course that this was in Paris, which is about 60m above sea level. A reasonable estimate of rho is in the neighborhood of 1.2. Most of the course is pretty smooth asphalt, though there are some slightly rougher spots. I was wearing arm warmers, leg warmers, and a wind vest, and I wasn't trying at all to be aero.

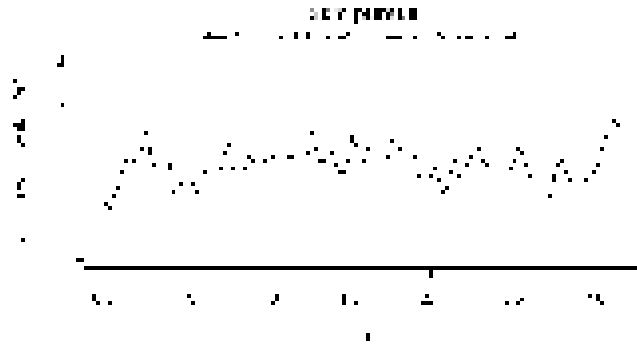
These data were not included with the challenge so I haven't discussed them. However, if you do use these data, along with a small amount of wind and an estimate of Crr of .0035, I get an estimated CdA for the first few laps of 0.415, and about .436 (5% higher) for the last two laps. It appears that with these data the method can discern a change in CdA smaller than that, down to 1%.

## flat TT



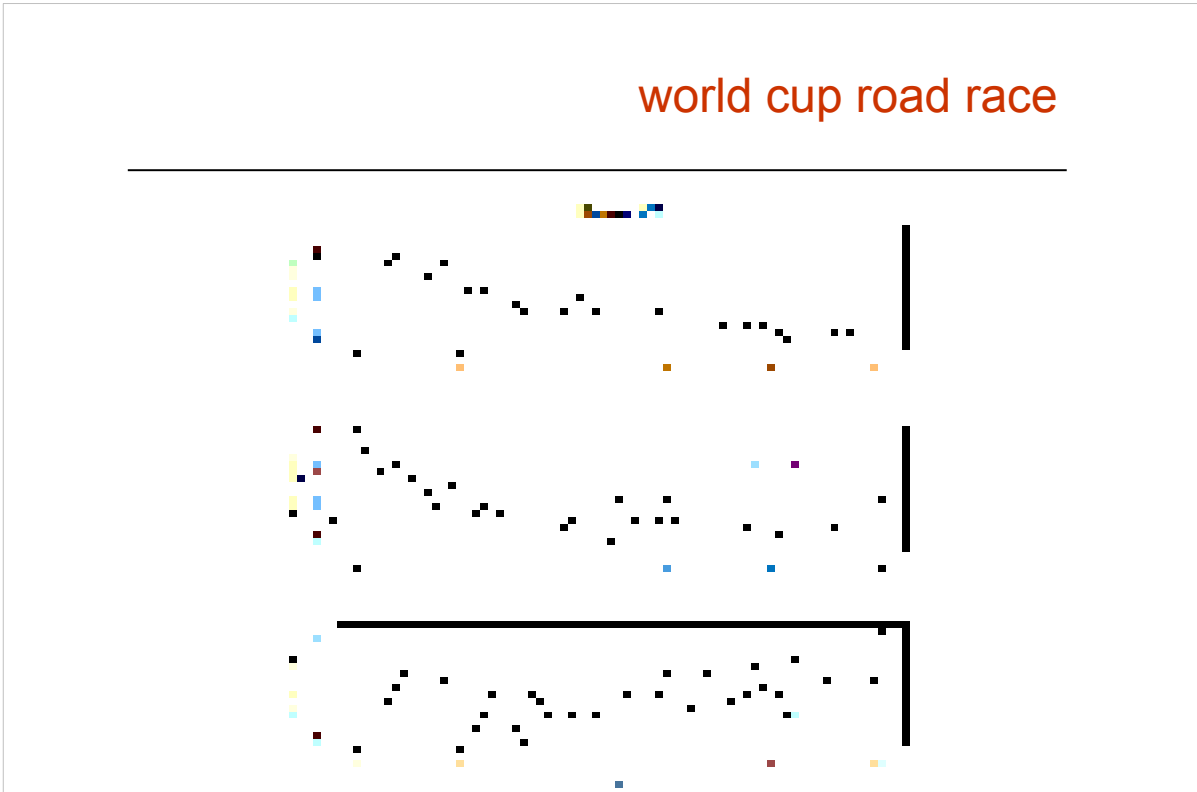
The data were collected by Kraig Willett, who posted these data on [biketechreview.com](http://biketechreview.com) with no other information than that the ride data were collected over three laps at Fiesta Island. The course is flat; reportedly, the actual elevation change is no more than a meter. Therefore, this virtual elevation profile can be interpreted as showing what the wind would have meant in terms of elevation. If you download Kraig's data and analyze it directly in terms of virtual wind you'll see it's much "noisier" than converting into virtual elevation. The profiles across laps are quite consistent, suggesting that the wind was quite steady. The CdA estimate appears quite low. In addition, it appears that he started off in the downwind direction. Subsequent to my analysis, Kraig said that the wind was blowing 7 or 8 mph, he was in aerobars and using 140mm BMX cranks, and made a guess at his weight that day.

## track race



The track race was a 3km individual pursuit. The data were collected by Bob Schwartz and posted on [rec.bicycles.racing](http://rec.bicycles.racing). The method shows 24 bumps or dips in elevation over the race, with most of the bumps appearing to fall within a one meter band (except at the very beginning and very end of the race). The dips are equally-spaced and represent the turns – this suggests that the race was held on a 250m track. After looking at the graph, I asked Bob about the track: it was the 250m outdoor Blaine/NSC track in Minnesota. Bob says that although he doesn't remember the exact total mass that day, he believes my estimate was low – Bob is a big guy. Fortunately, adding 10 kg to total mass has almost no effect on the estimated CdA, as we might expect for a flat race. Perhaps surprisingly, an increase in Crr of 33% decreases the estimate of CdA by only 2.5%. A 1% increase in rho decreases the estimate for CdA by about 1%, as we would expect. The data were collected at 2.52-second intervals, as for the data in the next example.

## world cup road race



Dede Demet's data file used to be posted on the Power Tap web site. I don't know her weight, the wind conditions, or how she used her brakes but we can still pick up the 12 laps of the race. This data set was the first I analyzed using this approach, in April 2003. Note the low estimated CdA, and the two different “clearing” values for the early part and the late part of the race. This was a hilly race and her mass matters much more to the CdA estimate – and I don't know what it is. However, it appears to have been low enough to suggest that she was sheltered in the peloton.

Demet's Power Tap was set on 2.52-second recording that day. In that mode the PT records only every other 1.26-second record. Accelerations were estimated across the 2.52-second records as if they were complete. As a check, I have deleted every other 1.26-second record from other data sets – the results appear robust for the data sets I've tried. This means that the short cut I used to estimate acceleration isn't too terribly critical. In fact, note that the calculations I've used for this method are relatively crude. The method appears to be moderately robust.



## virtual altimeter

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the next two examples show that the method appears to be both reasonably accurate and consistent

### up and down Palomar Mountain

black shows altitude as reported by an altimeter, red shows estimated altitude, emphasizing that braking is ignored

### two years of the San Bruno Hill Climb

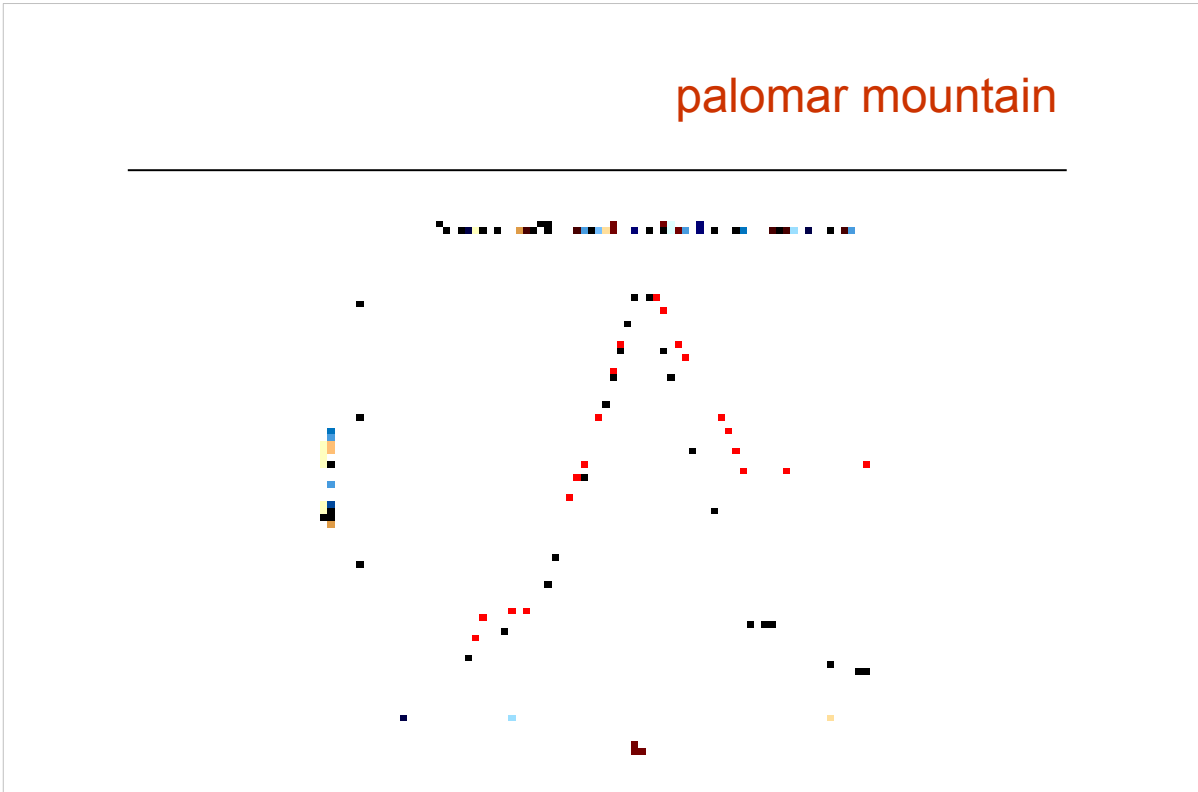
elevation profiles calculated from power and speed files for the same rider in two consecutive years, showing that the method captures consistent features of the profile

Both of these were previously presented at

<http://anonymous.coward.free.fr/wattage/altimeter/altimeter.html>

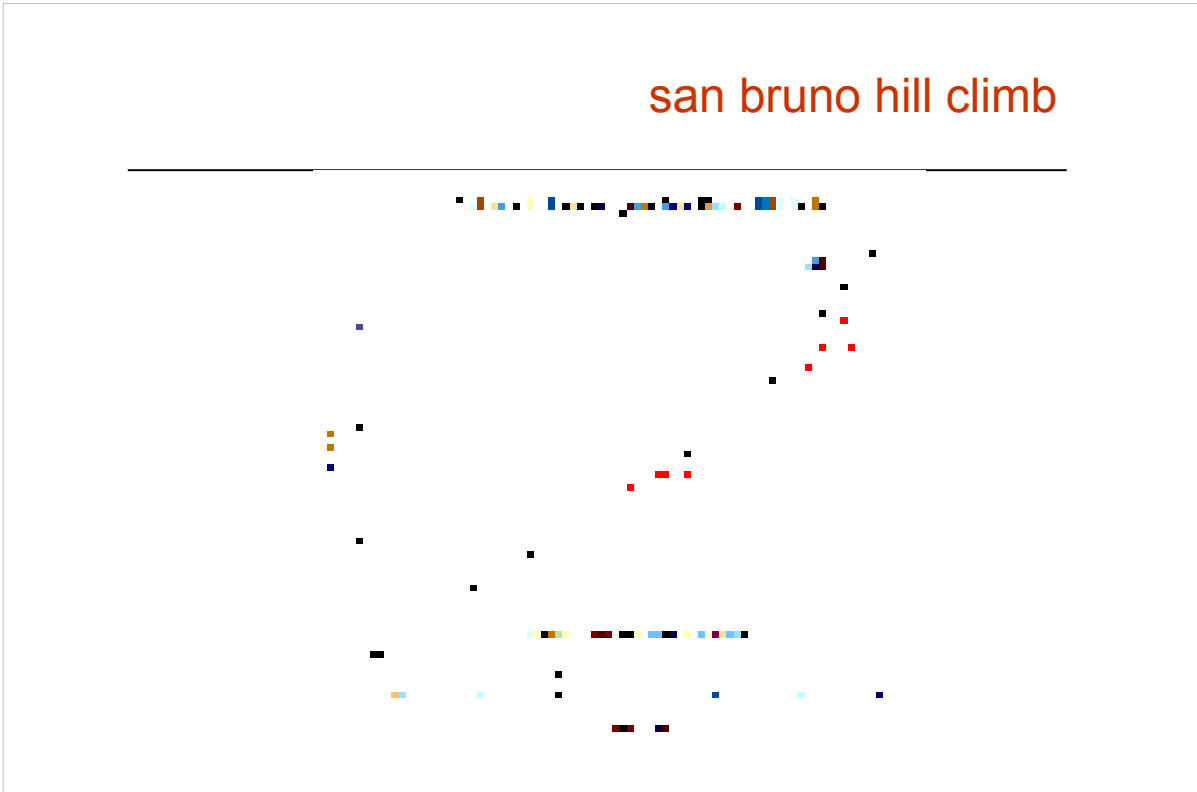
## palomar mountain

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The data were collected by Kraig Willett for his “three PM shoot-out.” The altimeter was the one on the Polar S710. The red line shows that the method works best when the brakes are not used, i.e., when power is well-modeled by the assumed speed-drag equation.

## san bruno hill climb



The data were collected by Gary Gellin during the 2002 and 2003 New Year's Day races up San Bruno Mountain, south of San Francisco, and posted on the Wattage list. Gary reported that his total mass was the same both years.

As an aside, note that although Gary's power was the same, his mass was the same, and the hill was certainly the same, his speed was ~6% higher in 2003 than in 2002. Thus, his VAM (velocità ascensionale media), or average ascent speed, differed by ~6%. VAM often gets discussed as a proxy for power and each year during the racing season naïve analysts will equate a value of VAM to a particular value for watts/kg. What this graphic shows is that a substantial difference in VAM can be observed for the same measured power (or, since Gary's mass was the same, in watts/kg). A 6% difference in observed VAM could mean, for example, the difference in the estimated power of a pro rider of between 6.0 watts/kg and 6.36 watts/kg.

## other examples

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- growing number of examples of this method being applied by other people on their own data
  - lots of flexibility on venue: out-and-backs, especially for “U”-shaped courses can be good
- spreadsheets exist to simplify calculation and an implementation is available in Golden Cheetah

Here are a couple of links that show other examples:

<http://alex-cycle.blogspot.com/2008/03/funky-things-with-power-meter-77.html>

<http://colinsbikingbits.blogspot.com/2009/09/time-trial-tri-bar-height-chung-test-3.html>

[http://forum.slowtwitch.com/cgi-bin/gforum.cgi?do=post\\_view\\_flat;post=1802183;page=1;mh=-1](http://forum.slowtwitch.com/cgi-bin/gforum.cgi?do=post_view_flat;post=1802183;page=1;mh=-1)  
;

<http://jasperga.blogspot.com/2009/11/chung-method-is-no-joke.html>

## functional precision

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- with practice, riders have reported consistently being able to distinguish small changes in position and equipment (like helmets, frames, or front brakes)
- as already discussed, VE profiles reveal previously undisclosed events like the momentary use of brakes or a change in position
  - riders have reported detecting a change when another rider enters a velodrome on the opposite side of the track, or a hill graded not to spec, or a client who had changed saddle height without informing his coach
  - This is a strong result:** VE profiles are functionally precise enough to make reliable predictions that are subsequently verified as true

As noted elsewhere, one rider attached a 5cm x 5cm (= .0025 m<sup>2</sup>) square to his bike and estimated the change in CdA at .0030 m<sup>2</sup>. Since a flat square has Cd slightly larger than 1, this is a good indication of the method's ability to detect and estimate small changes in drag.

Alex Simmons reports that while riding at the Dunc Gray Velodrome in Sydney he could determine a change in VE when another rider entered on the opposite side of the track. In addition, he has reported that while doing aero testing with a client, one of the VE runs just did not “fit.” On further investigation he learned his client had changed his seat height between runs. Both of these anecdotes suggest that when conditions are otherwise favorable the method has enough precision to identify small changes. This is a very strong result: the method detected a previously unknown or unobserved change that was subsequently found to be true. Note that in the original challenge data the method correctly identified previously undisclosed characteristics about the ride such as the number of laps, the momentary use of brakes at one point in the ride, and a change in position—and all of this was despite the data being flawed. I also spotted a consistent “dip” in the VE profile at the same location. On further investigation, a storm drain was located at the indicated spot.

## statistical precision

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- with careful experimental control by expert users, classic field testing has produced CVs on the estimate of CdA as low as 1.3% (e.g.,  $sd = .0025 \text{ m}^2$  on an estimated CdA of around  $.2 \text{ m}^2$ )
- direct assessment of the variability of VE-based estimates of CdA by “similarly” experienced users has produced CVs as low as 0.3% (i.e.,  $sd = .0006 \text{ m}^2$  on an estimated CdA of around  $.2 \text{ m}^2$ )

I'm not a huge fan of statistical inference in this context but I know some people are, and want to know more conventional measures of the precision possible with this approach.

The 1.3% claim for the precision of the classic method was made by Andy Coggan. He also has claimed that the 1.5% represents “typical repeatability” across different days of trials.

In general, I don't recommend comparing estimates across different days of trials since we can't usually ensure that the conditions are exactly comparable. VE test trials can take less time compared to classic regression tests, so often (depending on how long the laps or loops are and how long it takes to change configurations) you can do several configuration tests within an hour rather than on different days.

Tom Anhalt collected the data used to directly assess the variability of a VE estimate for his field comparison of the drag of two different front brakes. The R script used to analyze these data (and a pointer to the data themselves) is available at:

<http://anonymous.coward.free.fr/simkinsbrake/R/mach-egg.txt>

In that script, I use an analytic formula to find CdAs for all lap-length intervals in the data (there are about 500 of them and each lap-length interval has net elevation gain of zero). That approach (of assuming that each lap length interval zeros out in net elevation gain) depends on getting the lap lengths right and if you look closely at the graphs produced by that script you will see that the the last lap is "misaligned" by about 2 wheel revolutions compared to the previous two laps. That last lap is also the place where the estimated CdAs for the two different brakes coincide -- but for everywhere else, the two brakes are distinct. As expected with this type of approach, there is strong serial correlation in the estimates so I use Box-Jenkins time series techniques to remove the systematic component of the series and transform the estimates into a stationary process. The sd is then the square root of the variance of the stationarized estimates.

## do we need to know $C_{rr}$ ?

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- often, we don't
  - often (though not always) we're more interested in knowing how a change in position or equipment affects  $C_dA$ . If we test using the same tires and tubes on the same roads on the same day at the same pressure then  $C_{rr}$  is a constant and we can concentrate on estimating changes in  $C_dA$
- sometimes, we do
  - in those cases, we need a way to estimate both  $C_dA$  and  $C_{rr}$

Sometimes, even when you think you need an estimate of  $C_{rr}$ , you may not. For example, because the classic estimation approach is fairly time consuming, it's good to make a separate estimate of  $C_{rr}$  so you don't have to repeat a control run each time you go out. However, this method can be less time-consuming than the classic approach (sometimes, much less) so the need to get a separate estimate of  $C_{rr}$  is reduced. That is, the classic field test approach lets you make separate estimates of  $C_{rr}$  and  $C_dA$  but it requires multiple passes down the same course at a wide range of speeds. This approach will let you estimate *changes* in  $C_dA$  for a given  $C_{rr}$  (or changes in  $C_{rr}$  for a given  $C_dA$ ) much more quickly, especially if the laps are short and the range of speed within a run is wide. Quicker runs mean that doing a “control” run with a base configuration each time out isn't very onerous—and this is good experimental practice anyway.

Shorter laps have other advantages, too: they reduce the amount of time you're in the field so you less exposed to changes in conditions like changes in wind or passing cars.

As an aside, remember that in a wind tunnel, you don't get estimates of  $C_{rr}$ .

## prying apart CdA and Crr

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- we've been starting with a guess at Crr. Is it possible to estimate Crr separately?  
sometimes, yes
- first, understand why we've needed to do this  
the usual field test approach requires flat roads (or constant grade). The approach described here allows you to use roads that aren't flat so we've lost a constraint. We need some extra information to compensate.
  - one possibility: make known changes and repeat
  - another: know the true elevation profile

There are several methods that allow you to separately estimate Crr and CdA, depending on the data collected. Although I discuss one way here, if I were really interested in getting precise estimates of Crr I'd follow the protocol laid out by Tom Anhalt and used by Al Morrison for tire testing using rollers and a power meter. You can find the results of Al's tests at [biketechreview.com](http://biketechreview.com). Al's results are precise but the Crr on a real road can be higher, depending on the road surface. Nonetheless, it appears from informal road testing that the relative ranking of Al's roller tests is preserved.



## extra information demands extra care

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- requires very careful test runs
  - testing on a calm day helps a lot
  - knowing true elevation profile helps a lot
- two examples
  - same hill, different speeds
  - flat course, monotonic speed change (aka the Shen method)

We mean it: if you want to get precise results you really need to be precise in your data collection. Fortunately, this approach seems to be pretty robust in the sense that even if you don't precisely control everything it can produce reasonable ballpark results (unlike, as we saw earlier, the classic regression approach which can produce horrendous results if you're not careful). The flip side of the robustness is that you should avoid thinking that this method will work under any venue or weather condition whatsoever.

## same hill, different speeds

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- I rode up the same hill twice: once slow and once fast(er)
  - first ~ 170 watts, second ~ 245 watts
  - from topo maps climb known to be 114 meters
  - (this time) checked weight and air density
  - almost no wind
- thus far, we have only been solving for (Crr, CdA) pairs
  - if Crr overestimated then calculated CdA will be too low
  - if Crr underestimated then calculated CdA will be too high
  - more importantly, each (Crr, CdA) pair implies a different amount of total climbing
- knowing total elevation gain adds another constraint

The key is that a particular combination of Crr and CdA imply a certain range of elevation gain, i.e., in this case, we're not just "leveling" the lap profiles, we're also trying to match the total amplitude of the profiles.

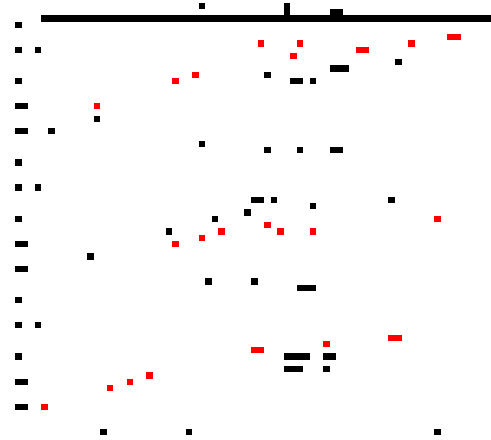
You can find the data file here:

<http://anonymous.coward.free.fr/wattage/cda/skylinefieldtest.csv>

As an aside, this ride wasn't intended as a data collection ride; it was only after I'd started the ride that I realized the conditions were essentially windless and I could use the ride for data collection. Intervals 1 and 3 are the climbing portions; interval 2 is the descent but the route is twisty enough that I had to use the brakes. The temperature was in the low 40's F, I was wearing tights and a winter jersey, and I estimated the weather conditions when I got back home after the ride.

## Crr and CdA constrain elevation gain

- for a given Crr and speed, you can always find a CdA that matches a given total elevation change
- but for two different speeds there is only one (Crr, CdA) pairing that matches a given total elevation change at both speeds. That's when the red and black lines coincide

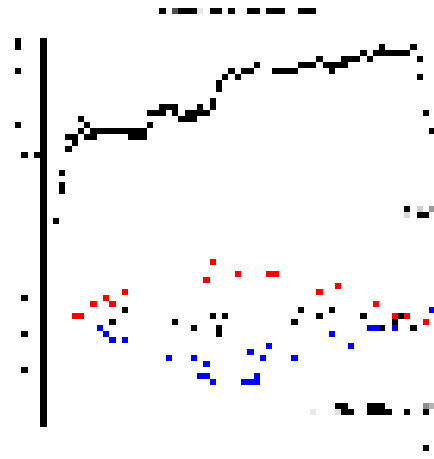


We can always find a “leveling” CdA for a given Crr but if the Crr is too low, the CdA will be too high and the VE profile will differ from the true lap profile; if the Crr is too high, the CdA will be too low and the VE profile will differ from the true lap profile *but in the opposite direction*. That's what we see here: in the top panel the red line (the higher power run) is above the black line; in the middle panel the red line is below the black line. Only one (Crr, CdA) combination produced the same total 114 meter elevation gain for both runs; that's what's shown in the bottom panel.

## flat course, monotonic speed change

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- increasing speed on a velodrome
- only one (Crr, CdA) pairing flattens both the overall profile and each of the individual lap profiles



Data collected by Andy Shen, and discussed here:

<http://velocitynation.com/content/coachingfitness/2009/shen-method>

## can we generalize?

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- Crr and CdA constrain total elevation gain – *but they also constrain elevation gain over any segment*
- if we know true elevation profile over the entire course we can fit to *arbitrary* segments

this can come in handy for velodrome laps since we know the true profile

be wary of using GPS altitude readings as the “true” elevation profile—consumer GPS units are accurate but may not be precise enough

be wary of using barometric altimeter readings as the “true” elevation profile—barometric altimeters are precise but may not be accurate enough

I've used a combination of barometric altimeter data with VABMs from topographic maps to “anchor” the altimeter profile. That can be very handy.

## twice around the block

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- not yet convinced? Try a quick “twice around the block” run  
find a nearby block that lets you make the turns **safely** (both for you and for others) without using the brakes. Start the interval recorder, ride around the block with your hands on the brake hoods then, still without braking, move your hands to the drops and ride around the block again. You can vary your speed as long as you can make the turns without hitting the brakes. Just hold your position and line. Then end the interval
- even with this crude test, as long as the wind isn't too terribly bad you ought to be able to see the change in CdA
- a couple of "twice around the block" trials will give you good insight into what you'd need to do to improve precision

**Always choose a safe venue.** No one cares to see your CdA carved on your tombstone.

You can pop the data into a spreadsheet like the one I pointed to earlier or you can download the data into Golden Cheetah which will have a module to do the calculations and graphing for you.

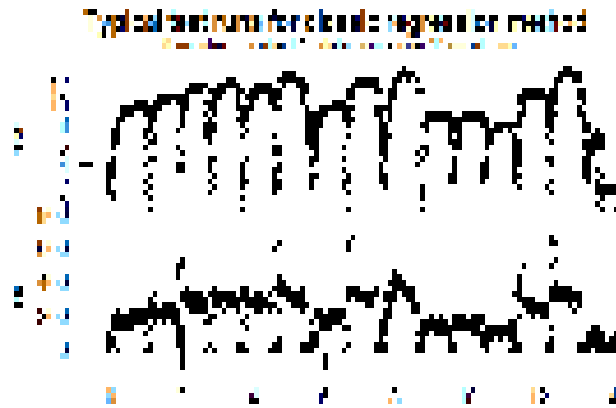
If the wind isn't too strong you ought to be able to see the “shape” of the block, and find an estimated CdA that puts you at the same virtual elevation at the start and end of the first lap. Then find an estimated CdA that puts you at the same VE at the start and end of the second lap. Simple. If you happen to have hit the brakes, you'll be able to see it as a jump in the virtual elevation at exactly the right location. Try it.

If you do happen to have to hit your brakes you can sometimes edit or snip out the little jump in VE in order to get the laps to line up. You don't want to do this a dozen times per lap (that gets old fast) but if there's a single stop sign on an otherwise perfect route this is a good way to salvage that route.

## revisiting the classic protocol

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- classic protocol uses multiple runs on flat wind-free venues each at a different speed. Speed and power might look like this:



These data were collected by Chris Mayhew on Centaur Rd (aka “Andy Coggan's wind tunnel.”)

## improving the classic protocol

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- instead, slowly and steadily increase from a stop up to your cruising TT speed, then coast down to 1 or 2 m/s. Repeat for inbound direction
- since we now know how to handle accelerations, calculate adjusted moment-by-moment power:  $w = \text{watts} - \text{watts}_{\text{KE}}$
- regress  $w$  on  $v$  and  $v^3$ 
  - this cuts the number of runs down which saves time while improving precision since sample size and resolution increases

During the coastdown try to soft-pedal.

If you know the slope, you can include that correction, too.

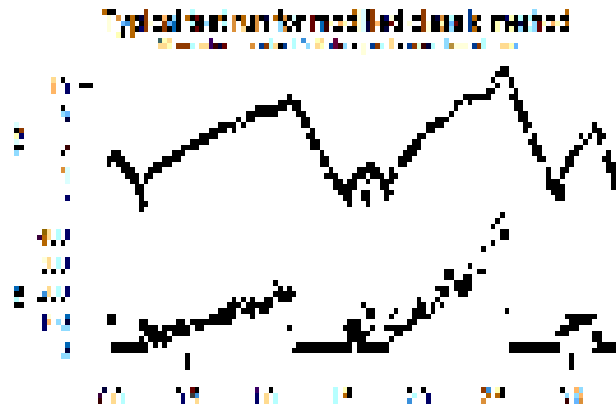
For the regression, include an outbound-inbound interaction (aka dummy) for the  $v^3$  term. If the interaction is significant you know something was different (typically, wind) between the outbound and inbound runs. Using a dummy interaction in this way improves precision compared to either “pooling” the outbound and inbound runs or to doing separate outbound and inbound regressions.



## faster, shorter, better

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- speed and power might look like this:



These data were collected on the almost flat Bay Trail on a rare day when the wind was almost zero. Notice the coasting segments where power was zero? The VE formula works whether power is zero or non-zero.

## estimating CdA without a power meter

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- coast downs are common method to estimate CdA
  - some compare top speed, others compare rollout
  - some use hills, others use flat roads
- usual coast downs have poor precision
- can get improved estimates with good speed recording
  - VE is based on power and speed; in coast downs you know power is zero so record speed with a data logger and proceed as usual
- possible to do this with bikes, cars, soapbox racers, racing wheelchairs

Ingenious people have developed several variants of coast downs but few have investigated precision.

Repeated coast downs with a data logger to record speed is essential if you want precision. You'll need to do control run(s) to assess precision. An inexpensive datalogger is described here: <http://www.hupi.org/HPeJ/0012/0012.html>, but many riders are now using GPS head units with their bikes that record speed. If you have a GPS unit, use a wheel-based speed sensor (usually, a low-cost option) to get higher speed precision than is available by GPS location alone.

A precision coast down method is described here:

<http://www.xs4all.nl/~cp4trml/metingen/measurements.html>

Aero drag testing of racing wheelchairs with coast downs is discussed here:

<http://www.ae.illinois.edu/racingwheelchair/Coastdown.html>

Note that in the two latter links, the authors thought that a non-flat venue was a problem and interfered with getting precise estimates.

Automobile “ecomodders” have been doing conventional coast downs in order to estimate CdA. They could improve their estimates with a higher-precision speed logger.

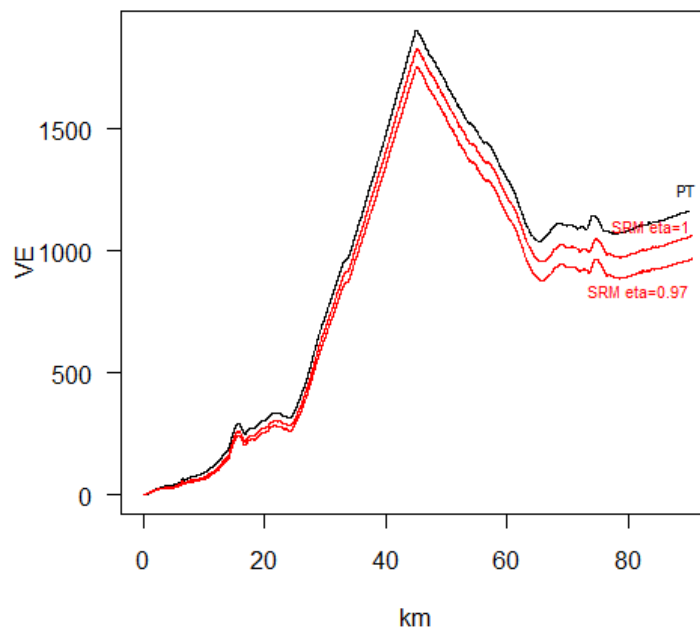
## applications

- racing wheelchairs
  - in 2007 UIUC Dept of Aero Eng sponsored wheelchair racing club and performed aero drag tests in concourses underneath football stadium using conventional methods. Poor precision because floor was uneven. High-precision speed recording combined with VE could produce better results
- soapbox derby racers, go karts, drivetrain losses, etc.
  - Martin et al. (2006) presumes fixed drivetrain loss of 2.3%. VE could be used on a bike equipped with both a SRM and PT: find the difference in recorded powers that makes SRM and PT VE profiles identical

Perhaps surprisingly, recumbent and hpv builders rarely measure the drag of their vehicles—neither aero, nor rolling resistance, nor drivetrain losses.

Here is a comparison of VE from a ride up Mt. Palomar during which ride data were recorded with a PT, a SRM, and a Polar power meter. Crr, CdA, as well as all other parameters except eta, the drivetrain loss, were assumed to be the same. Note the eta that must be used to get the SRM and PT to come close to the same VE profile is greater than 1.0.

**Calibration and drivetrain efficiency: PT vs. SRM**



## shallow hill coastdowns

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- here are data from two coastdowns I did one afternoon on a shallow hill. The speeds are in km/h, at one second intervals
  - run 1: 15.2, 15.8, 16.3, 16.9, 16.9, 17.4, 17.7, 18.1, 18.5, 18.9, 18.6, 17.9, 17.9, 18.6, 19.7, 20.0, 20.9, 21.6, 22.5, 22.5, 23.4, 23.7, 23.5, 24.3, 25.1, 25.7, 26.0, 25.6, 25.3, 24.6, 24.2, 23.9, 23.5, 23.1, 22.9, 22.6, 22.3, 22.3, 22.0, 21.9, 21.7, 21.8, 21.4, 20.7, 20.9
  - run 2: 26.6, 26.6, 26.6, 26.5, 26.6, 26.8, 26.1, 25.4, 25.6, 26.3, 26.8, 27.3, 27.9, 28.4, 29.1, 28.7, 29.6, 29.8, 30.6, 30.3, 29.7, 29.2, 28.7, 28.3, 27.4, 27.2, 26.7, 26.1, 25.8, 25.5, 25.3, 25.0, 24.5, 24.2, 23.9
- coast downs = zero power. Let's assume total mass = 86 kg,  $\rho = 1.17 \text{ kg/m}^3$ , and total drop from entry to exit of the test section was exactly 5 meters. Can you estimate CdA and Crr from these data?

The answer is: Yes, you should be able to estimate CdA and Crr. The estimated CdA is .037, the Crr is .0066. Note: this was on my commuter bike and I was wearing street clothes so don't whine about my parachute-like CdA and boat-anchor-like Crr.

Many riders who don't have power meters do have a way to record speed using something like one of the Garmin units.

To solve this problem, use the closed-form work-energy equations given earlier. Make substitutions for the total mass, the air density, and the change in elevation given above. You will have two equations for two unknowns (CdA and Crr). If you do extra runs (and you should) you could use regression to find minimum loss estimates for CdA and Crr.

If you do choose this protocol, use a shallow hill so you can start and end your coastdowns at different speeds – a very steep hill will mean that you reach terminal velocity on both runs, while a shallow hill will let you maintain a difference in speed across the runs over the entirety of the test section. That will help improve the precision of your estimates.

When you have the estimated CdA and Crr, create VE profiles as a visual check to see how well they match each other, and the “true” profile of the road.

## summary

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- knowing only speed and power still provides an impressive amount of information when data are collected over laps
  - with these data, small changes in CdA are estimable
  - it's possible to examine how these estimates are affected by air density, wind speed, and wind direction
- knowing speed, power, and a little about the course provides even more information
  - you can tune the model not only to line up the profiles but also to match total elevation gain
- in some cases, knowing a bit more info can help you to get separate estimates of Crr and CdA (if that's what you need to do)

Precision in the speed and power measurements is key. If you don't have precision in those two then all bets are off. (Note that for coastdowns, power is fixed at zero – presumably, you'd know that with some precision). If you're trying to identify small changes in drag then you'll need either precision in the measurement of wind or else you'll need to test on a wind-free day.

## main conclusion

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- up to now, if you had an on-bike power meter, most field test protocols required that you find a flat venue and perform tests at constant speed
- **Using this approach eases data collection and greatly expands the number of appropriate field test sites you can use**
  - you can get good diagnostics of when a test run didn't work so you can improve your experimental technique
  - model deviations are localized so you can spot them
  - precision is no worse and can be better than classic method
- wind is still a problem
  - but diagnostics provide a clue for when to worry and you can sometimes “crop” your data to exclude short interruptions (such as a passing car)

Expanding the number of appropriate field test venues is key. As observed earlier, if you have an otherwise perfect test venue (sheltered from the wind, low traffic, smooth surface) marred by a single stop sign, you can often “snip out” the discontinuity in the VE profile making it usable.

VE test runs often take less time than classic regression runs so you can do more configuration tests in a single session and don't have to make comparisons across test days.

When carefully done by an experienced user, the precision of VE estimates using ideal protocols has been shown to be superior to classic regression estimates. In any event, since VE methods work on constant speed and slope data (i.e., the data collected for classic methods) at the very worst their precision will be the same as classic methods.

## what kinds of venues work?

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- flat road or velodrome, as for “classic” approach? *yes*
- industrial park loop? *yes*
- single out-and-back up a slight hill? *yes*
- twice up the same hill at different speeds and power? *yes*
- long residential block shaped like a “half-pipe” with a dip in the middle? *yes*
- almost any wind-sheltered route where you don't use brakes? *yes*  
    *ideal venue could be a bowl-shaped route that lets you speed up and still slow down at the ends to make the turnaround*
- city street with stop signs, wind gusts, and passing traffic? *nope*

**Always choose a safe venue.** Don't make your turns or turnarounds in places where you don't have good sight lines.

Some of these will require slightly different test protocols than others. For example, if you do an out-and-back you may need to “re-set” the virtual elevation at the turnaround if you brake.

In the “classic” approach you need to get a wide range of speeds and you get that range of speeds by doing lots of runs each at a constant speed. If you use a flat “classic” venue with this approach you don't need to hold speed constant but it's still a good idea to get a wide range of speed. A little bit of elevation change lets you do that pretty easily, and a bowl-shaped route lets you get that speed and then scrub it off before you need to make the turnaround so that's a preferred solution – but in a pinch you can use a flat venue if that's what you have.

If you want precision choose a venue that's wind-sheltered and has little traffic. This method is sensitive enough that a passing car can screw up that portion of a run for several seconds. Even a passing cyclist can show up as a “blip” in the VE profile.

If you don't know the true elevation profile (which most of us don't) then it's a good thing to vary your speed a fair bit both across and within runs. Having some elevation change helps you do that, with the obvious constraint that you don't want to use your brakes.

If you do want to know the true elevation at certain spots, it's handy to know that cities often rely on gravity-fed sewer systems so they precisely measure the exact elevation of manhole openings. Check with your city or utility district.

## recommendations

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- use the “modified classic” method if your venue allows. If not, do laps or loops
  - shorter laps let you do more of them
- don't hold speed constant
  - the wider the spread across laps the easier it is to isolate separate effects
    - a small amount of elevation change can help increase speed variation as long as it's not so steep you need to brake
- measure air density, don't use your brakes if you can help it, and if you're trying to detect small changes do this on a wind-free day, or use an on-bike wind meter

VE precision is responsive to the amount of effort you put in, i.e., it's possible to get okay results with surprisingly sloppy technique but if you want good precision you have to do things precisely. If you care about precision and you haven't done so already, do a static calibration check on your PM to make sure it's reading right.

For your first few runs your experimental technique will suck but practice makes perfect and even with sucky technique you may be surprised at how good your results are. Nonetheless, as you get more experience your results will get better and better.

Even though you'll want to do this on a windless day, you can scout out routes and practice how to hold your line on less-than-perfect days.

Hold your position steady and make sure you follow exactly the same line through the turns or turnarounds. Sometimes a few chalk marks come in handy to help you hit your line the same way each lap. Just a couple of little X's should suffice. Try the first laps at (relatively) high speed and that way you'll know you can do all subsequent laps at a lower speed and still hold your line.

You don't need to be obsessive about hitting the lap interval marker on your computer—taking your hands off the bars will mess up your aero position and VE will be able to pick out your laps anyway or, if it doesn't, it means the conditions weren't right. Just hit your interval marker a few seconds before you begin the test and then when you're all done.

I often toss the first lap; that's one of the advantages of doing short laps. Even with practice, my first lap is often noisier than subsequent laps.

If you're not using a PT, you'll want to make a correction for drivetrain losses.

Let your PM come to ambient temperature and re-zero it before you start the test runs. If you do a practice lap and lay down some chalk that should be plenty of time before zeroing.